

Organisers' report on the workshop "Torsors: theory and applications"

10-14 January, 2011

There was no deviation from the original proposal.

Short description

The idea of the meeting was to bring together mathematicians who use torsors in their research. Two mini-courses presented recent achievements on the borderline between geometric invariant theory and representation theory. One morning session was devoted to new applications of étale homotopy; arithmetic applications of torsors featured in a number of other talks. On the whole, talks were accessible to a heterogeneous audience of experts, postdocs and PhD students coming from different backgrounds (algebra, algebraic geometry and arithmetic geometry). We had a good mix of speakers: well established influential researchers as well as brilliant younger people. Much discussion went on between the talks. One goal has already been achieved: mathematicians working in areas that are related but which had limited contact have become aware of the existing methods and techniques. We are confident that this cross-fertilisation of ideas will continue. There was sufficient enthusiasm to publish a volume of conference proceedings featuring lecture notes of the two mini-courses.

Comprehensive report

The aim of the conference was to bring together those who work with torsors from different perspectives and those who apply torsors in arithmetic geometry. We can roughly break the talks into four groups:

- 1) geometric invariant theory and algebra of torsors,
- 2) representation theory,
- 3) arithmetic geometry,
- 4) algebraic geometry.

The centrepiece of the first group was the mini-course by Jürgen Hausen as well as talks by Ivan Arzhantsev, Antonio Laface and Serguei Gayfullin. Hausen's very well prepared and carefully delivered mini-course was an excellent introduction to modern developments in geometric invariant theory, in particular what concerns the actions of algebraic tori on varieties, to the Cox rings and their links to universal torsors. Many participants found it very useful. Arzhantsev's interesting talk contained quite a few new results at the interface of geometric invariant theory and representation theory.

The second group consisted of the mini-course by Vera Serganova and the talk by Philippe Gille. The use of Lie algebras and their representations is a novel development that led to a uniform proof of Batyrev's conjecture by Serganova and Skorobogatov, linking the universal torsors over del Pezzo surfaces to the projective homogeneous spaces of simple Lie algebras. The mini-course introduced the key constructions from Lie theory and explained how they are applied to the

geometry of del Pezzo surfaces. Open questions and fascinating new perspectives of research were discussed.

The third group included a variety of talks on a broad range of subjects. We had a survey talk by Victor Batyrev featuring his new conjectures. Number theoretic applications of torsors played a central role in Felipe Voloch's energetic talk which presented his exciting result on the finite descent obstructions for integral points on modular curves. This work has promising links to modularity and the Fontaine-Mazur conjecture, and can potentially lead to very strong results about rational points on curves. Jacob Stix gave a report on his recent work on Grothendieck's anabelian conjectures. Ulrich Derenthal explained his proof of a new case of the Batyrev-Manin conjecture on the number of points of bounded height. David Harari talked about a generalisation of the theory of descent (originally due to Colliot-Thélène and Sansuc) to open varieties, and its applications to integral points (joint work with Alexei Skorobogatov). Cyril Demarche gave a powerful talk on strong approximation on homogeneous spaces of algebraic groups. He explained very nicely the many ingredients of his work, in particular, his calculation of the Brauer group of torsors and homogeneous spaces of algebraic groups.

An entire morning session was devoted to the applications of étale homotopy theory of Artin and Mazur to rational and integral points and the Brauer-Manin obstruction. All three leading experts in this area were present at the conference (Ambrus Pal, Tomer Schlank, Yonatan Harpaz). Their talks gave a comprehensive view of this novel subject, which provides a clear and uniform picture of the hierarchy of obstructions to rational and integral points, including the original Brauer-Manin obstruction, its étale version, and the variants for open varieties. Many participants have expressed their interest in this new area, and pointed out how instructive this series of talks was.

Finally, the fourth group consisted of talks by Jenia Tevelev, Matthieu Florence and Mikhail Borovoi. Florence's talk contained a sketch of proof of his important recent result on the rationality of quotients of Grassmannians by the actions of tori, based on an explicit construction of a section for this action. Borovoi talked on his joint work with the late Joost van Hamel.

This varied programme was met with much approval by the participants. Since the audience of the workshop was expected to be heterogeneous (experts, postdocs and PhD students coming from various backgrounds such as algebra, algebraic geometry and arithmetic geometry) the speakers were instructed to give talks mainly accessible to this mix of people. It is fair to say that the speakers did a particularly good job. The participants found most of the talks really useful and instructive, and appreciated their very high scientific level. One goal of the conference has already been achieved: mathematicians working in related and yet not so much interacting areas have become aware of the existing methods and techniques. We are confident that this cross-fertilisation of ideas will continue.

There were active discussions between the talks. Although several leaders of their respective fields (such as Jean-Louis Colliot-Thélène) chose not to speak, their presence was conducive to lively interactions. In their answers to the questionnaire, quite a few participants indicated that they continued their existing collaborations at the workshop (Hausen, Arzhantsev, Laface and Derenthal worked on their book in progress, Colliot-Thélène and Skorobogatov worked on their paper in progress, as

did Hausen and Herppich). They also indicated that they are likely to start new collaborations.

A volume of conference proceedings featuring lecture notes from two mini-courses, and contributions by other participants is in preparation. We expect that it will also contain two long papers on étale homotopy, which will make it a unique source of information on the state of the art in the theoretical aspects of rational points and surrounding mathematical theories. A tentative agreement to publish it in the LMS Lecture Note Series has been reached.

The participants much liked the ICMS premises (including the conference room and the computer room) and have found the new location of the ICMS excellent. One participant mentioned that it would be nice to have more toilets available. The wine reception, the guided tour and the conference dinner were very lively and well attended, and helped the participants to get to know each other across their subjects boundaries.

All participants praised the work of the ICMS staff before and during the conference. Questions were answered and problems were solved. One participant whose luggage was lost by an airline was particularly grateful to the ICMS staff for doing all the work and making sure that the luggage was delivered to the participant. The conference web site was helpful and regularly updated. The organisers and the participants alike very much appreciated the work of Helene Frossling who has been incredibly helpful and efficient.