

International Centre for Mathematical Sciences
Research-in-groups

Singular solutions of mathematical elasticity in compound domains and modelling of failure in bi-material structural interfaces

Summary report:

The project has led to a novel collaborative work involving an interdisciplinary research team, which includes scientists from Italy and UK, as follows: Prof. Gennady Mishuris (Aberystwyth University, Leader of the Project), Prof. Davide Bigoni (University of Trento, Italy), Dr. Andrea Piccolroaz (University of Trento, Italy) and Prof. Alexander Movchan (University of Liverpool). The group has also had a productive interaction with Prof. Natasha Movchan (University of Liverpool) during the course of the work. A postgraduate student Mr Luca Argani (funded by the University of Trento, Italy) has visited Edinburgh, at no extra cost, to interact with members of the research group.

The duration of the work was one month, and all objectives set in the original proposal have been fully covered. The main tasks include:

- RO1. Analysis and solution of matrix functional equations of the Wiener-Hopf type corresponding to nontrivial singular solutions of the Lamé system in a space with 3-dimensional interfacial crack.**
- RO2. Asymptotic and numerical modelling of waves and fault propagation in media containing finite thickness structural interfaces with emphasis on localization of the physical fields within the interface.**
- RO3. Solution for the mechanical field produced by a dislocation or a dislocation dipole in a prestressed, elastic material.**

The progress made can be outlined as follows:

RO1: Factorisation of the Wiener-Hopf equation is the most important part completed. Furthermore, a special class of singular solutions (both symmetric and skew-symmetric), known as weight functions, have been analysed to study asymptotics of stress and strain near the crack edge on the interface. A range of applications in asymptotic problems of crack-defect interaction have been developed.

RO2: The problem has been reduced to a functional equation of the Wiener-Hopf type, which has been analysed in detail. Applications are in problems of dynamic contact within structured media. The analytical work has been accompanied by a series of numerical simulations for the local energy release associated with propagation of a non-symmetric fault within a structured interface.

RO3: In the classical literature, the theory of dislocations in solids has been developed mainly for elastic materials, unloaded in their natural state. We have extended the theory to cover the possibility that the material be prestressed and to analyze the consequences of the prestress. This extension requires the introduction of an incremental formulation based on the nominal stress, along directions previously explored for only different problems, for instance, bifurcation of nonlinear solids. We have started from a solution given by Eshelby of a dislocation in linear elastic solid and we have extended these ideas to incremental nonlinear elasticity. The solution displays a singularity at each dislocation edge, and a special effort has been directed toward the analysis of the case where one of the singular points moves to infinity, which is a generalization of the edge dislocation solution of linear isotropic elasticity. The solutions obtained here reveal features of the behaviour of severely deformed metals near the shear band formation threshold.

During the research period, started on July 5 and ended July 30, the team worked continuously at the ICMS in an exceptionally stimulating research environment, allowing several interactions with scientists visiting the centre. The team had the opportunity of actively participating in 17th Congress of the European Society of Biomechanics held in University of Edinburgh during 5 - 8 July 2010, where Prof. Mishuris gave a scientific presentation. Moreover, the team had the possibility of visiting the D'Arcy Thompson Museum at Dundee University, a great opportunity to enrich the knowledge of biological structures of interest in biomechanics.

All the work proceeded on schedule and within the planned budget.

Currently, we are working on two research papers, which will be submitted for publication in international journals in Mechanics and Applied Mathematics. We are grateful to ICMS for the support and for provision of excellent academic environment. This support will be gratefully acknowledged in all publications and lectures resulting from the current work.