# ICMS – Research-in-groups

## International Centre for Mathematical Sciences

## Research-in-groups

**Project title**: **Fractional derivative operators involving various special functions and applications of them at studying boundary-value problems for fractional order singular partial differential equations**

## Team members:


Team leader: Michael Ruzhansky (Imperial College of London, UK)

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## Summary Report:

The duration of the project was two weeks in the period 15-31 July 2016. The project was originally devoted to the analysis of the evolution processes described by an operator involving fractional time derivatives with Bessel-type space singularities. The results obtained during the research have significantly exceeded the original plans shaping a future research program devoted to a more comprehensive analysis of more general classes of operators of this type.

Two main approaches to the fractional Cauchy problems have been outlined:

1. Using the spectral decompositions of the space-part of the operator the analysis of the Cauchy problem can be reduced to a fractional ordinary differential equation. Consequently, one obtains a representation formula for solutions that can be used to derive a number of their properties. This approach has been analysed in the literature in the following cases:

- diffusion-type operators, when the orders of the fractional derivatives do not exceed one;
- a single fractional derivative of (any) order;
- the space-counterpart of the operator having specific form.

The approach developed during our project allows one to extend the applicability of this method by using more involved representations of solutions based on multinomial Mittag-Leffler functions together with the global Fourier analysis developed recently in the context of the

general analysis of boundary value problems[i]. As a consequence it would allow coupling processes described by fractional derivatives with different types of behaviour in space variables.

2. Another approach to fractional evolution equations consists in the extension of the classical Green function techniques to the setting of fractional derivatives. This approach has been successfully implemented for fractional telegraph equations of certain type where the Green function has been also known in an explicit form[ii]. In the project we have shown how the Green function method can be extended to prove the well-posedness of fractional evolution equations of higher orders as well as for the space part of the operator involving singularities, such as those appearing in Bessel-type operators.

In addition to these, a number of further interesting advances have been made, including:

- relations between known representations of solutions in terms of Mittag-Leffler functions with Srivastava-Daoust hypergeometric functions;
- new estimates for multinomial Mittag-Leffler functions;
- finding conditions for the solvability of the inverse problems for fractional derivatives Bessel evolution equations.

The team has started work on several research papers presenting results obtained in this project. The support of the ICMS has played a crucial role in allowing the team to conduct the joint collaboration. It will be gratefully acknowledged in the papers.

During the stay at ICMS, team members had a chance to attend some lectures given at the conference 'Symplectic geometry and topology' that took place at ICMS during 25-29 July 2016.

The directions for further joint research have been identified and plans have been made on a possible joint monograph by the team members devoted to the analysis of fractional derivative evolution equations.

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[i] M. Ruzhansky and N. Tokmagambetov, Nonharmonic Analysis of Boundary Value Problems**,** *Int Math Res Notices (2016), Vol. 2016, 3548-3615.*

[ii] M. Mamchuev, Boundary value problems for equations and systems of equations with partial derivatives of fractional order, KBSC, Russian Academy of Sciences, Nalchik, 2013.