RISK ANALYSIS AND PRICING OF LONGEVITY-LINKED SECURITIES

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PLAN FOR TALK

- Background and the mortality problem
- Stochastic mortality models
- Parameter and model risk
- Market-consistent pricing
- Closing remarks

The Problem

2007: What we know as the facts:

- Life expectancy is increasing.
- Future development of life expectancy is uncertain.

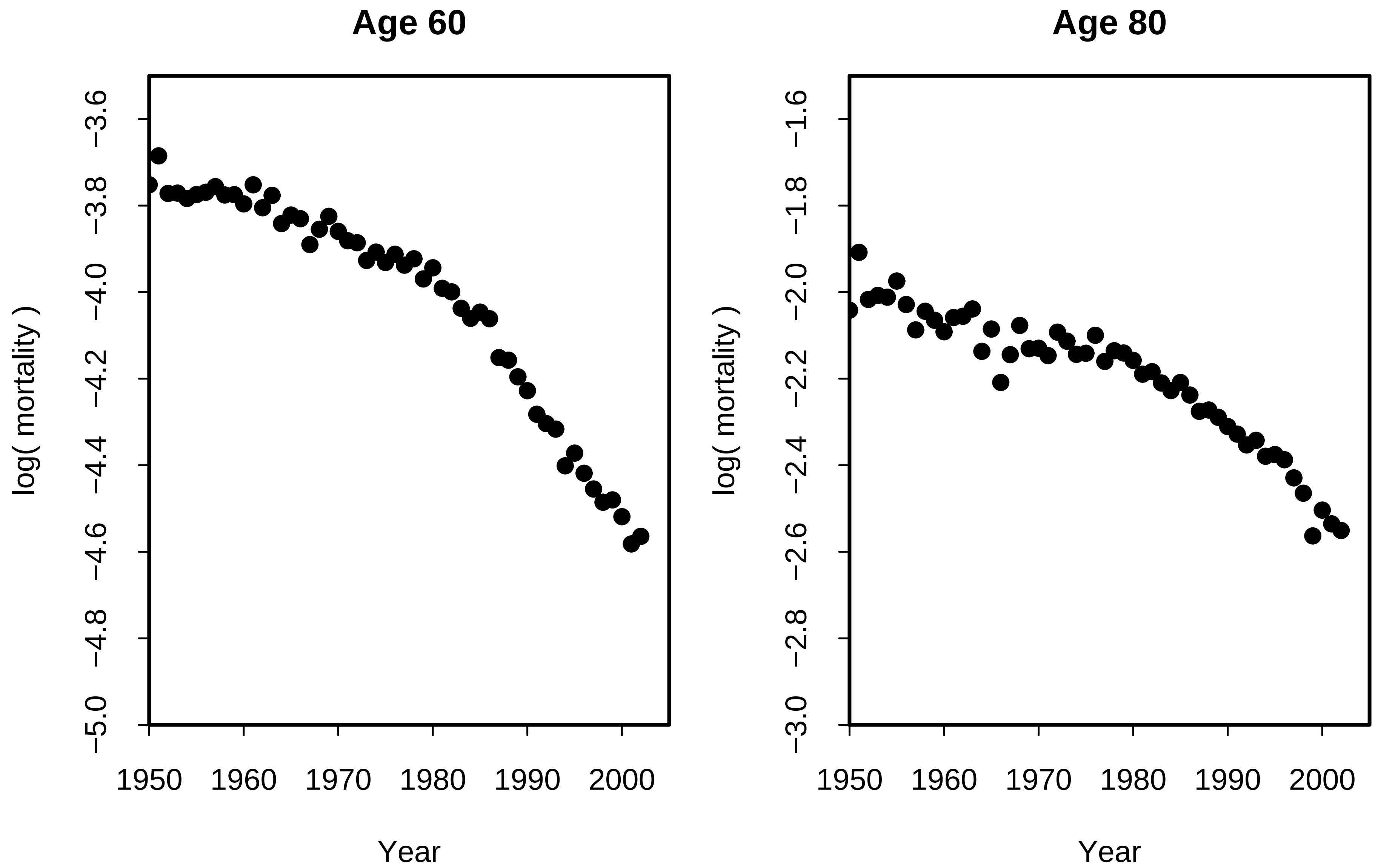
"Longevity risk"

$\Rightarrow$ Systematic risk for pension plans and annuity providers

The Problem – UK Defined-Benefit Pension Plans:

- Before 2000:
  - High equity returns masked impact of longevity improvements
- After 2000:
  - Poor equity returns, low interest rates
  - Decades of longevity improvements now a problem

England and Wales log mortality rates 1950-2002



Why do we need stochastic mortality models?

Data $\Rightarrow$ future mortality is uncertain

- Good risk management
- Setting risk reserves
- Life insurance contracts with embedded options
- Pricing and hedging mortality-linked securities

Stochastic mortality

- Many models to choose from
- Limited data $\Rightarrow$ model and parameter risk

How to compare stochastic models (*)

- Quantitative criteria
- Qualitative criteria
  - parsimony and transparency
  - robust relative to age and period range
  - biologically reasonable
  - forecasts are reasonable

(*) Cairns et al. (2007) A quantitative comparison of stochastic mortality models.... Online: www.lifemetrics.com

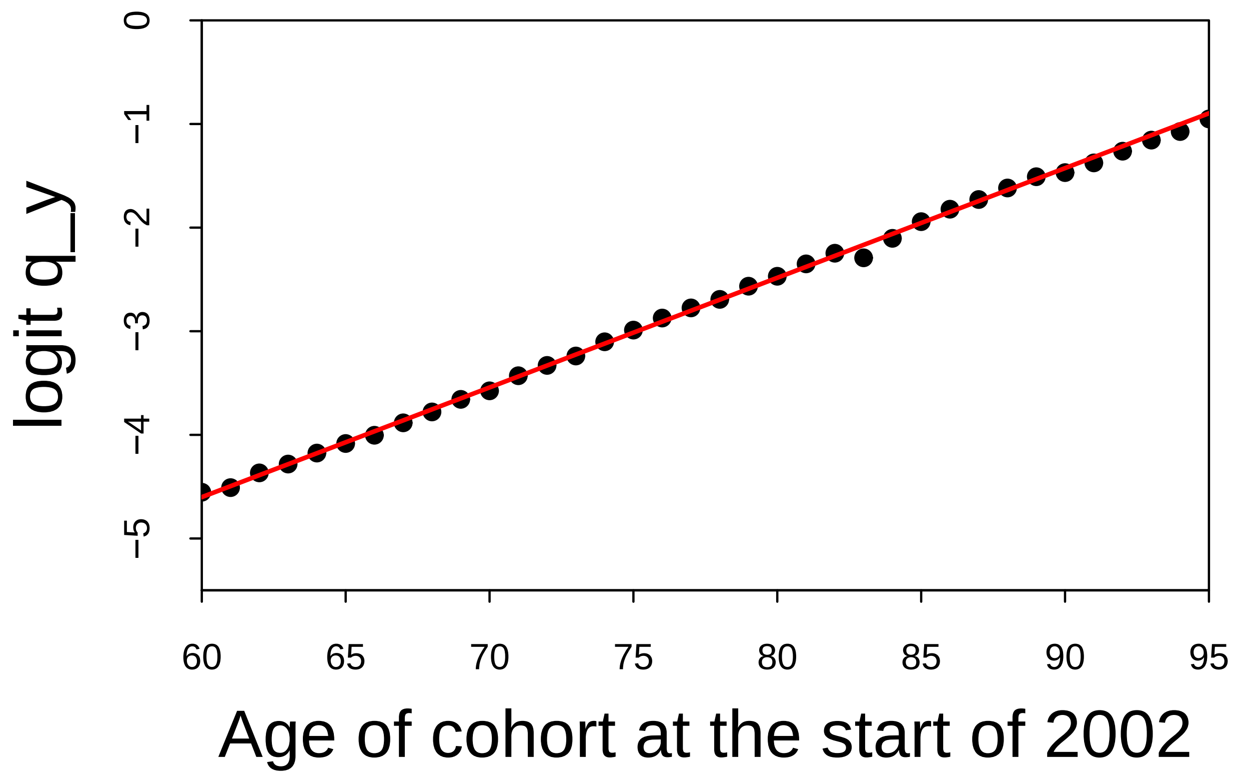
A TWO FACTOR TIME-SERIES MODEL

- $q(t,x) =$ mortality rate in year t at age x
- Mortality model for higher ages

Cairns-Blake-Dowd Model (CBD Age-Period model):

$$\log\left(\frac{q(t,x)}{1-q(t,x)}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x})$$

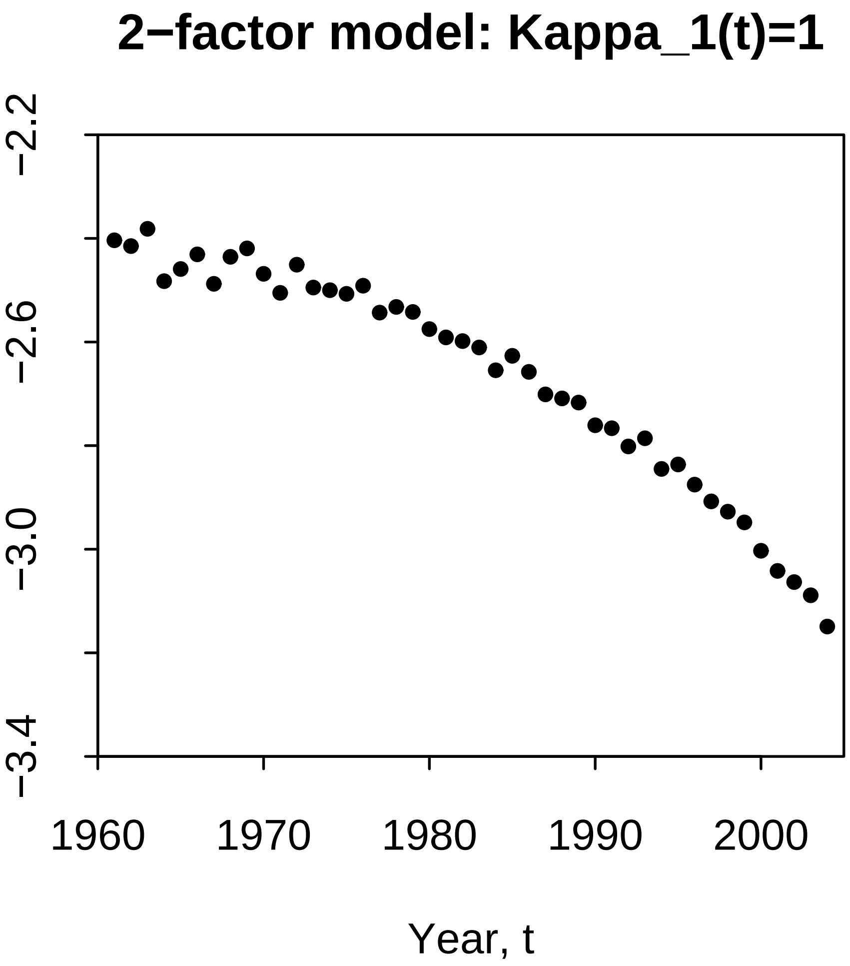
CBD Model: Case study – England and Wales males

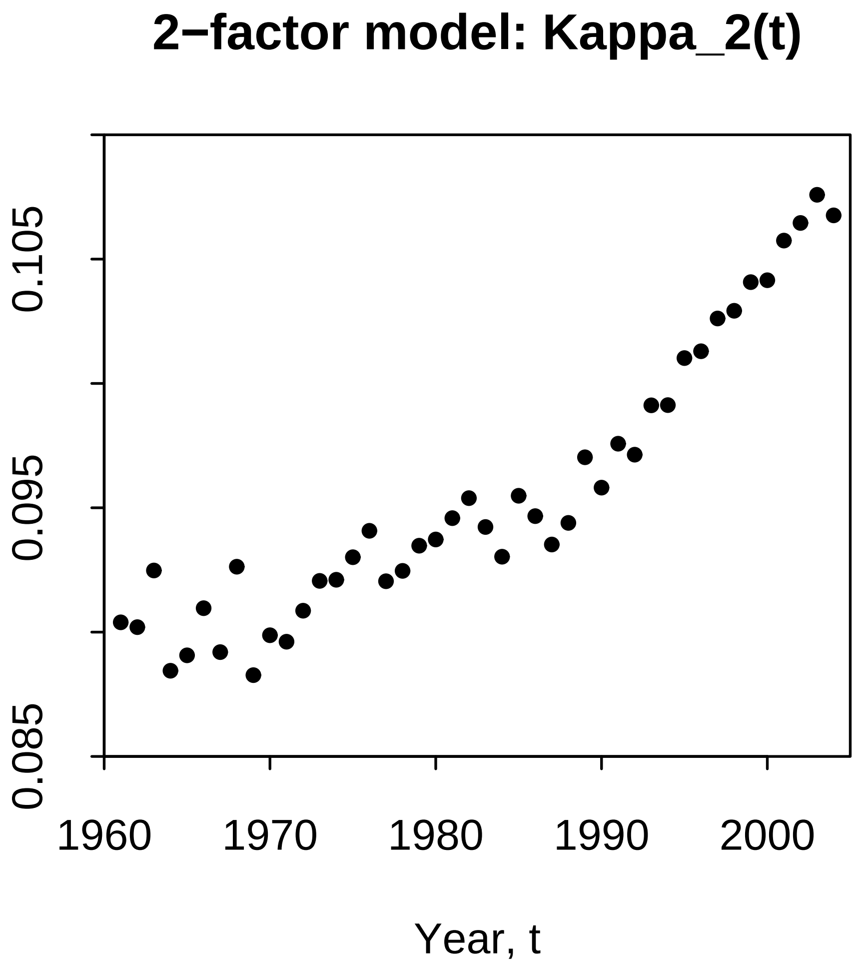


$\kappa_t^{(1)} \Rightarrow$ level

$\kappa_t^{(2)} \Rightarrow$ slope

CBD Model


2−factor model: Kappa_1(t)=1
−2.2
−2.6
−3.0
−3.4
1960
1970
1980
1990
2000
Year, t


2−factor model: Kappa_2(t)
0.105
0.095
0.085
1960
1970
1980
1990
2000
Year, t

$\kappa_t = (\kappa_t^{(1)}, \kappa_t^{(2)})'$

Model: Random walk with drift

$$\kappa_{t+1} - \kappa_t = \mu + CZ(t+1)$$

- $\mu = (\mu_1, \mu_2)' =$ drift
- $V = CC' =$ variance-covariance matrix
- Estimate μ and V
- Quantify parameter uncertainty in μ and V

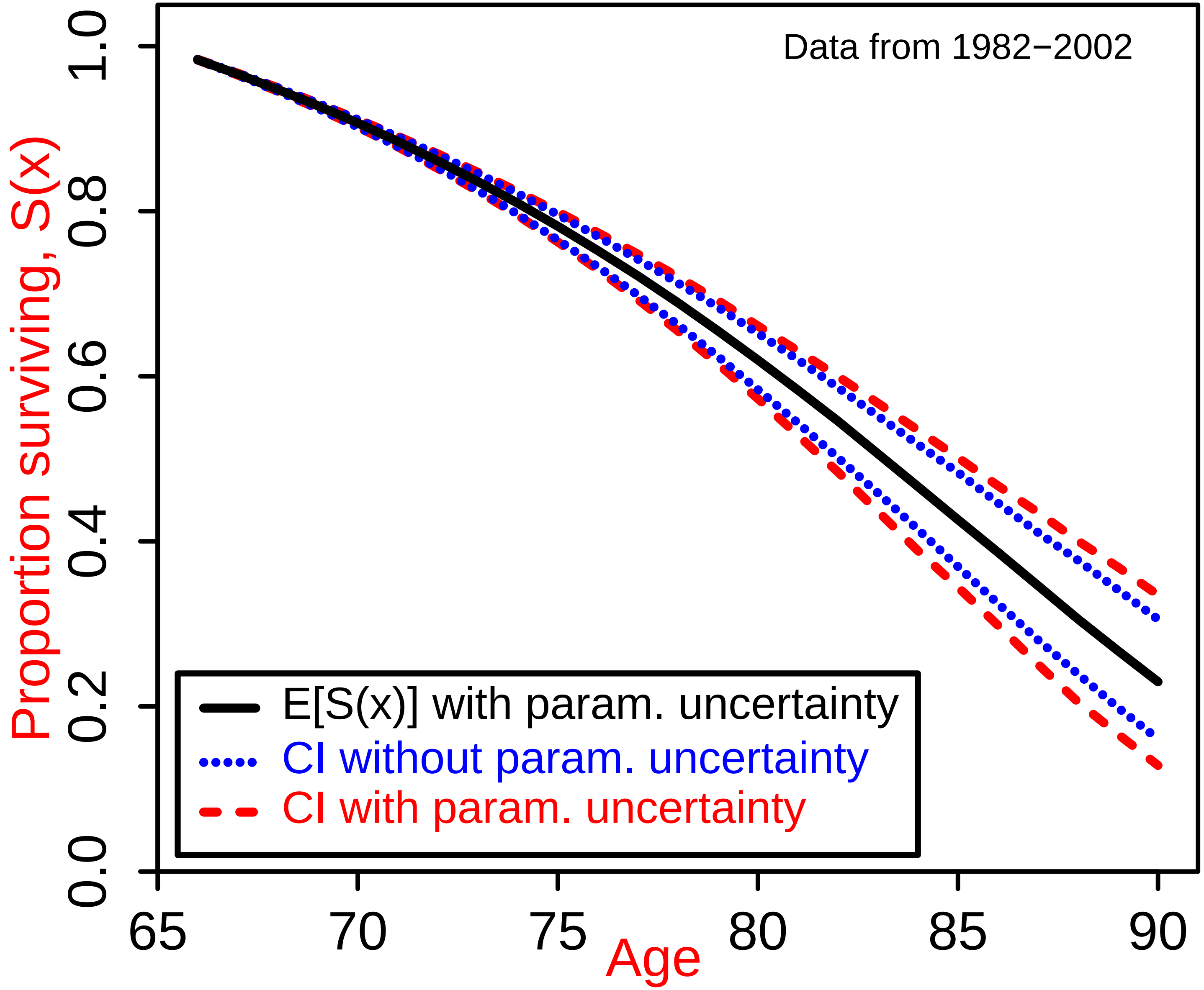
Application: cohort survivorship

- Cohort: Age x at time $t = 0$
- $S(t, x) =$ survivor index at t

  proportion surviving from time 0 to time t

$$S(t, x) = (1 - q(0, x)) \times (1 - q(1, x + 1) \times \ldots$$
$$\ldots \times (1 - q(t - 1, x + t - 1))$$

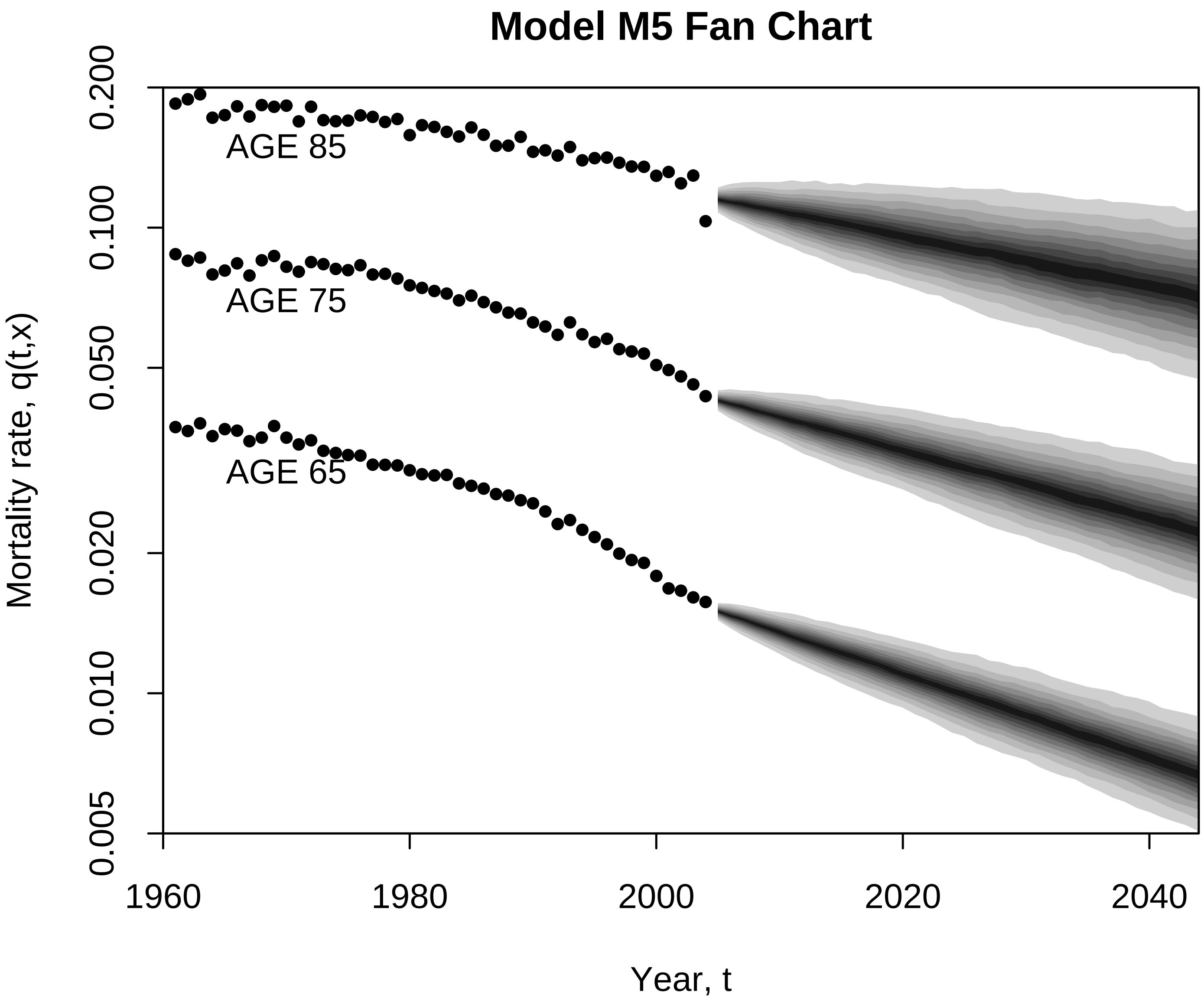
90% Confidence Interval (CI) for Cohort Survivorship



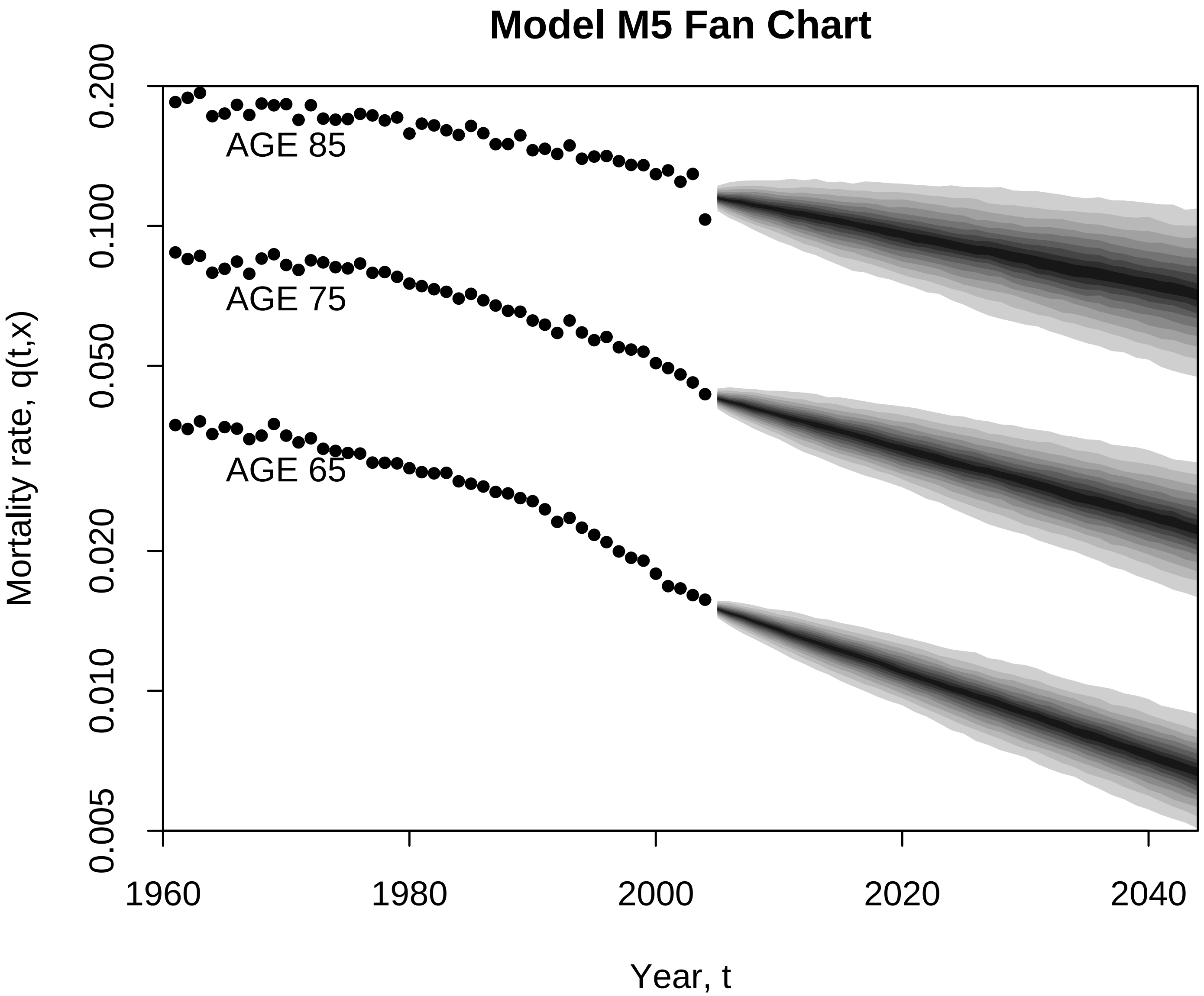
Cohort Survivorship: General Conclusions

- Less than 10 years:
  - Systematic risk not significant
- Over 10 years
  - Systematic risk becomes more and more significant over time
- Over 20 years
  - Parameter risk begins to dominate

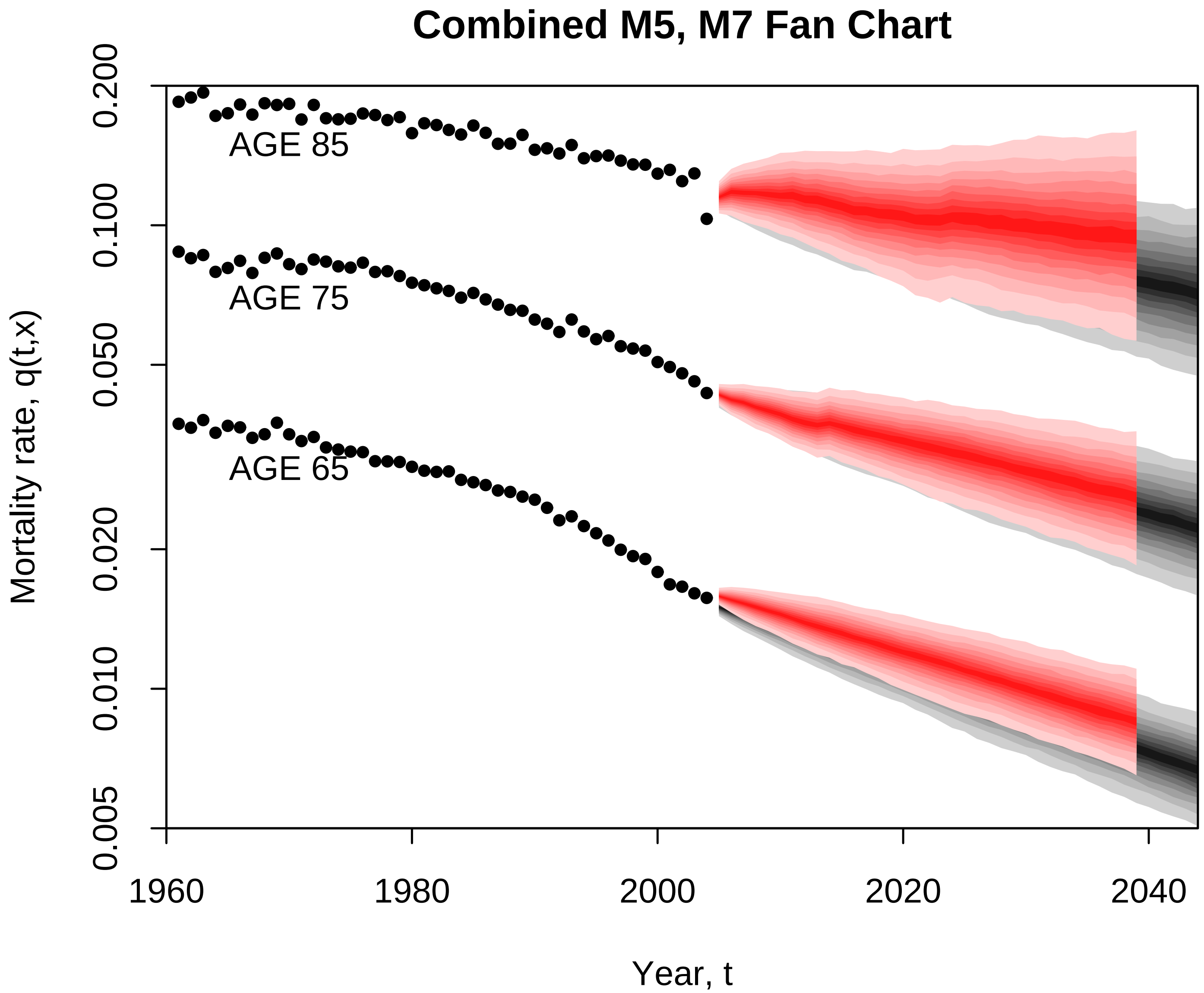
Mortality Fan Charts + A plausible set of forecasts



Model risk

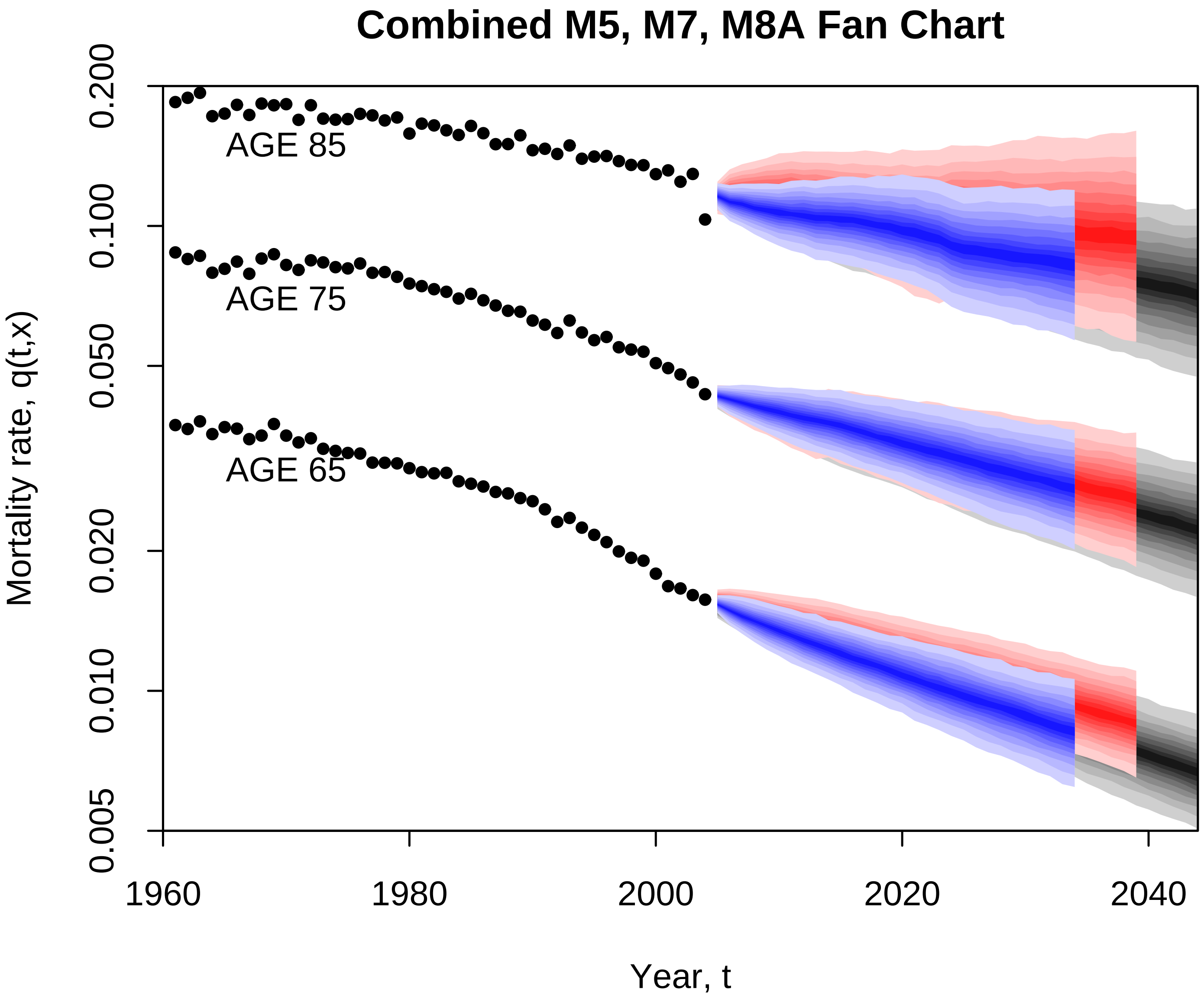

Model M5 Fan Chart
Mortality rate, q(t,x)
0.200
0.100
0.050
0.020
0.010
0.005
AGE 85
AGE 75
AGE 65
1960
1980
2000
2020
2040
Year, t

Model risk



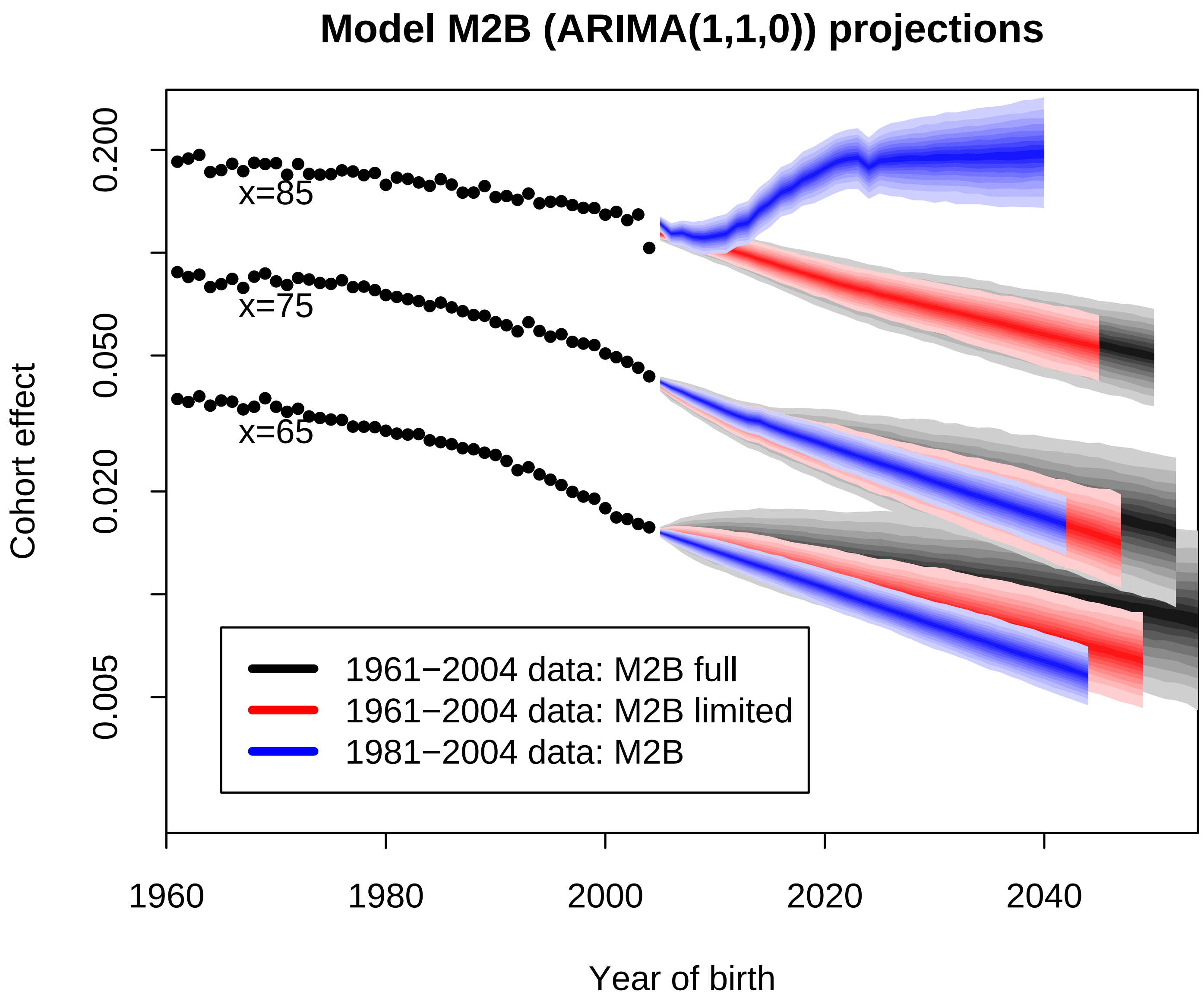
Combined M5, M7 Fan Chart
AGE 85
AGE 75
AGE 65
Mortality rate, q(t,x)
0.200
0.100
0.050
0.020
0.010
0.005
1960
1980
2000
2020
2040
Year, t

Model risk



Combined M5, M7, M8A Fan Chart
AGE 85
AGE 75
AGE 65
Mortality rate, q(t,x)
0.005
0.010
0.020
0.050
0.100
0.200
1960
1980
2000
2020
2040
Year, t

Not all models are robust: Renshaw-Haberman model



Application: Pricing a Longevity Bond

- e.g. EIB-BNP longevity bond
- Very simple contract
- Bond pays 50M $\times S(t,x)$ for $t = 1, 2, \ldots, 25$
- $x =$ initial age of reference cohort

How do you price a longevity bond?

- Hedgers are prepared to pay a premium
- Two approaches:
  - Take *real-world* expected values

    use a risk-adjusted discount rate
  - Take *risk-adjusted* expected values

    use the risk-free discount rate

Risk-neutral pricing (risk-adjusted expected values)

$$\kappa_{t+1} \;=\; \kappa_t + \mu + CZ(t+1) \qquad \text{(original model)}$$

$$\downarrow$$

$$\kappa_{t+1} \;=\; \kappa_t + \mu + C\left\{\tilde{Z}(t+1) + \lambda\right\} \qquad \text{(pricing)}$$

$$\tilde{Z}(t+1) = \Big(Z_1(t+1), Z_2(t+1)\Big)', \quad \lambda = \Big(\lambda_1, \lambda_2\Big)'$$

- $\tilde{Z}_1,\ \tilde{Z}_2$: risk-neutral $\sim iid\ N(0,1)$
- λ_1 and λ_2: market prices of risk

How does the market price of risk work?

- Two independent sources of risk $Z_1(t)$, $Z_2(t)$
- Tradeable security has corresp. volatilities σ_1, σ_2
- Market price of risk is

  the additional expected return over the risk free rate

  per unit of risk
- Hence

$$\text{Risk premium} \quad = \left(\sigma_1\lambda_1 + \sigma_2\lambda_2\right)$$

Comments

- The market is highly incomplete
- The switch from P to Q is <u>a modelling assumption</u>
- (Simple) Key assumption:

  market prices of risk λ_1 and λ_2 are constant.
- As a market develops this assumption becomes a testable hypothesis

$\leq$ One data point: the EIB-BNP longevity bond

- Offer price (ultimately unsuccessful) $\Rightarrow$

  average risk premium of 20 basis points

  (paid by the buyer of the bond to the seller)

  if held to maturity

- What values of λ_1, λ_2 are consistent with the 20b.p.'s risk premium?

- One price, two parameters $\Rightarrow$ many solutions

Answer: 20 b.p. spread equates to

$$\lambda_1 = 0.375, \qquad \lambda_2 = 0$$

$$\downarrow \qquad\qquad \downarrow$$

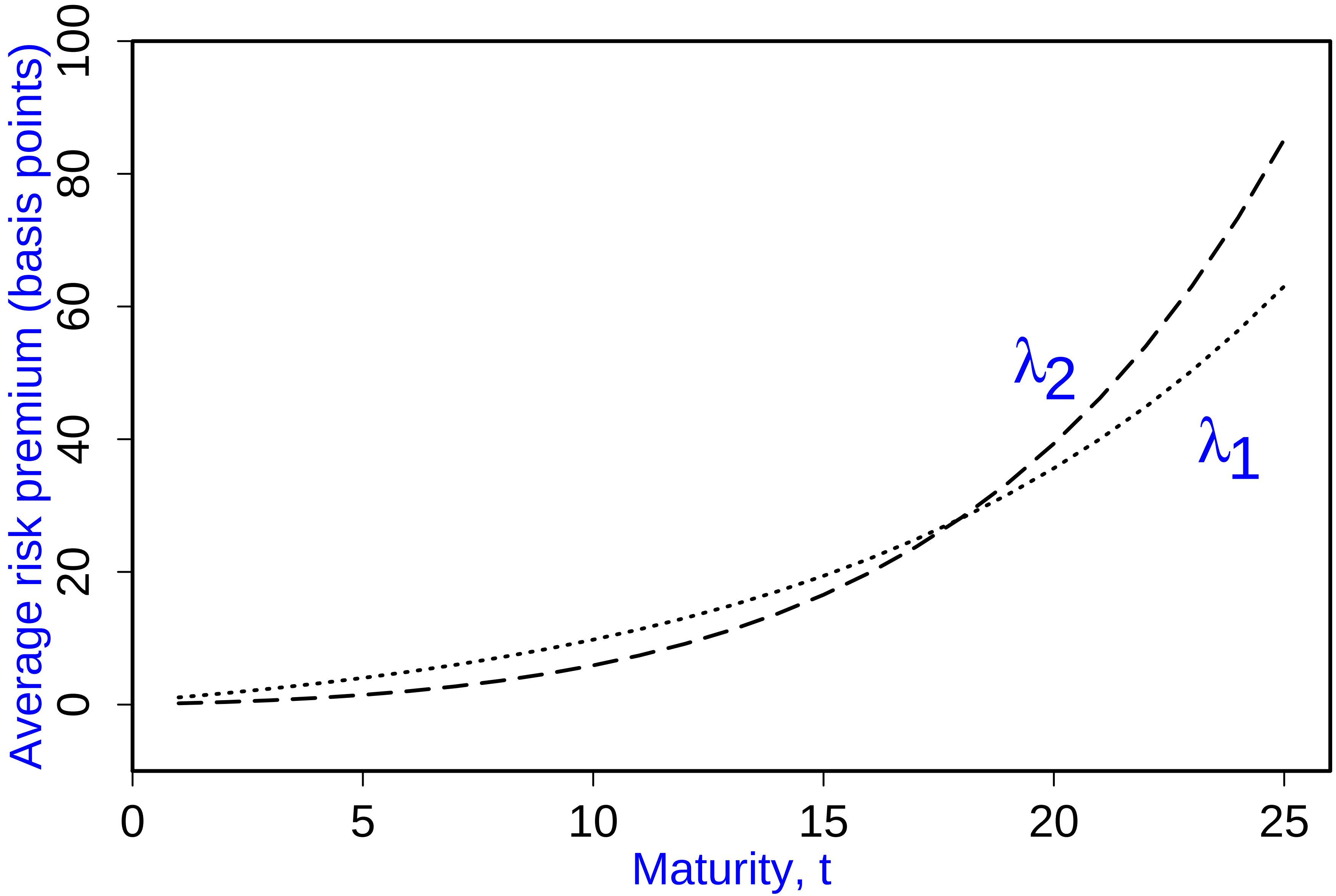
$$\lambda_1 = 0, \qquad \lambda_2 = 0.315$$

Do these values represent a *good deal*?

Why do we need to know λ_1, λ_2?

$\Rightarrow$ info. on how to price new issues in the future.

Zero-coupon Longevity bonds: avg. risk premium p.a.



Longevity Bond Risk Premiums: $\lambda = (0.375, 0)$

Dependency on term and initial age:

		Initial age of cohort, x		
		60	65	70
Bond	20	8.9	14.7	23.1
Maturity	25	12.7	20.0	28.7
T	30	16.9	24.3	31.5

Closing Remarks

- Parameter and model risk cannot be ignored
  - Extensive testing reveals considerable insight
- Range of new models under development
  - Cohort effects
  - Market models
- Risk neutral approach to pricing:
  - Simple and consistent approach to pricing risk

References

- Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Belevich, I. (2007) A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. Preprint.
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