Correlation and Diversification in Integrated Risk Models

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“Principles not Rules”

Contents

1. Correlation and Diversification

"Quantitative standards. Again more questions - Do the risk measurement systems capture potentially severe tail events achieving a soundness standard of 1 in 200 for a one year period? Do the risk management systems capture the major drivers of risk affecting the shape of the tail of the loss estimates? Are the systems for measuring risk correlations sound, implemented with integrity and take account of the uncertainty surrounding any such correlation estimates, particularly during periods of stress. Are the correlation assumptions validated using appropriate quantitative and qualitative standards?" [Tiner, 2006]

Accounting for Diversification

> "Solvency I fails to recognise diversification benefits properly even though they are fundamental to value creation in the insurance industry and contribute to improved efficiency of insurance service provision, greater stability in financial performance which in turn contributes to policyholder protection, and a more efficient allocation of capital in the economy." [Treasury and FSA, 2006]

The pooling of risks across portfolios, business lines, organisations achieves diversification. The extent of the diversification benefit depends on the degree of dependence between the pooled risks. Aggregate solvency capital should reflect the diversification benefit.

Layers of Aggregation

In [Kuritzkes et al., 2002] three levels of aggregation are identified:

1. stand-alone risks within a single risk factor (e.g. underwriting risk in each contract of a domestic motor portfolio);

2. different risk factors within a single business line (e.g. combining asset, underwriting and operational risks in non-life or life insurance);

3. different business lines within an enterprise.

Mathematical Framework

An enterprise may be split into d sub-units (business lines, risk factors by business line, contracts/investments). Each sub-unit generates a loss or a (negative) change-in-value L_i over the time horizon of interest. The aggregate change-in-value distribution is given by

$$L = L_1 + \cdots + L_d \, .$$

Ideal goal:

Determination of risk capital should be based on a stochastic model for $(L_1, \ldots, L_d)$ that accurately reflects the dependence structure.

Diversification and Correlation

> " Diversification benefits can be assessed by correlations between different risk categories. A correlation of $+100\%$ means that two variables will fall and rise in lock-step; any correlation below this indicates the potential for diversification benefits." [Treasury and FSA, 2006]

The last statement is not true of ordinary linear (Pearson) correlation! But true of rank correlation.

Lock-step

The mathematical term for this is comonotonicity. It means all risks are increasing functions of a common underlying risk: $(L_1, \ldots, L_d) = (v_1(Z), \ldots, v_d(Z))$. Such risks would be considered undiversifiable.

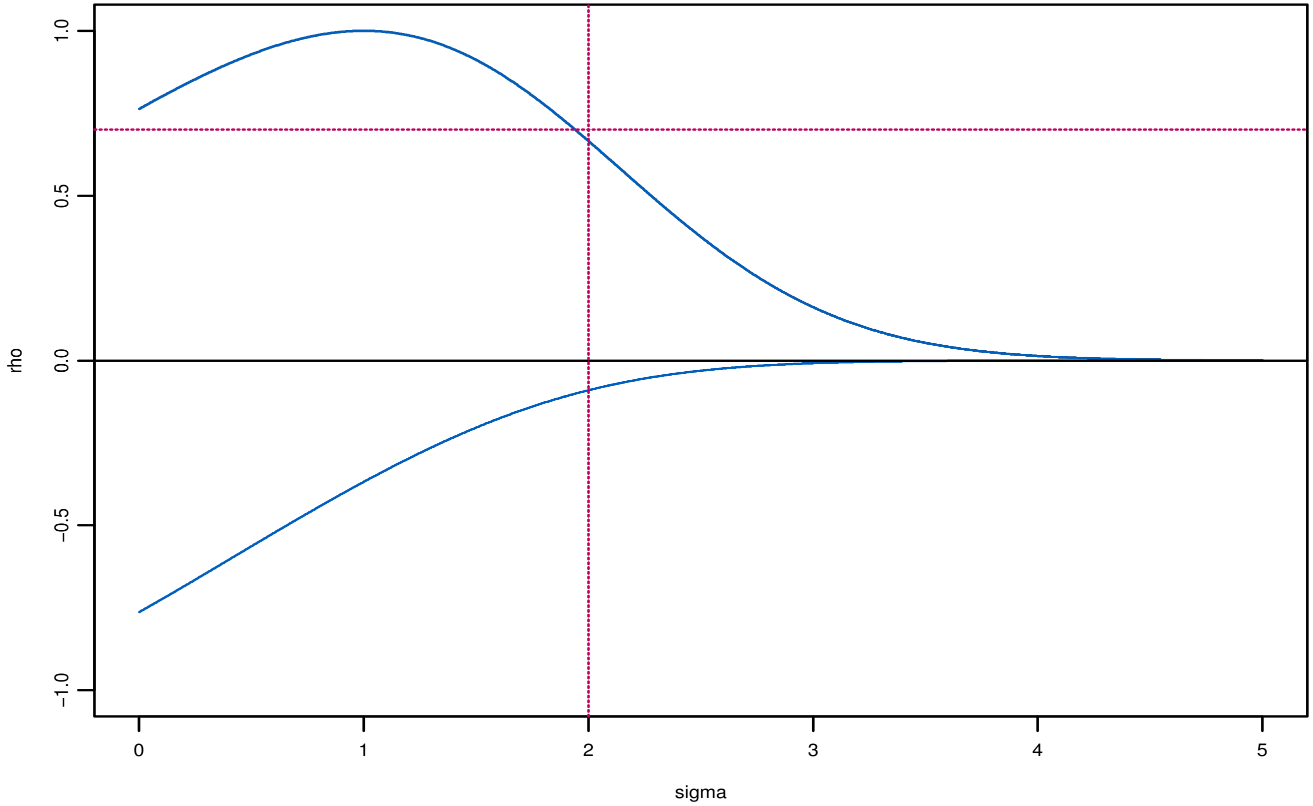
Comonotonicity and Correlation

$$(\text{linear correlation} = 1) \;\Rightarrow\; \text{comonotonicity}$$

$$\text{comonotonicity} \;\not\Rightarrow\; (\text{linear correlation} = 1)$$

- We can create models where individual risks move in lock-step (are undiversifiable), but have an arbitrarily small correlation.
- For two given distributions, attainable correlations form a sub-interval of $[-1, 1]$.
- Upper bound corresponds to comonotonicity, lower to countermonotonicity (negative lock-step)
- Our intuition about linear correlation is in fact very faulty!

Example of Attainable Correlations



Take $X_1 \sim$ Lognormal$(0, 1)$, and $X_2 \sim$ Lognormal$(0, \sigma^2)$. Observe how interval of attainable correlations varies with σ. Upper boundary represents comonotonicity. See [McNeil et al., 2005] for details.

Correlation Confusion

"Among nine big economies, stock market correlations have averaged around 0.5 since the 1960s. In other words, for every 1 per cent rise (or fall) in, say, American share prices, share prices in the other markets will typically rise (fall) by 0.5 per cent."

The Economist, 8 November 1997

"A correlation of 0.5 does not indicate that a return from stockmarket A will be 50% of stockmarket B's return, or vice-versa...A correlation of 0.5 shows that 50% of the time the return of stockmarket A will be positively correlated with the return of stockmarket B, and 50% of the time it will not."

The Economist (letter), 22 November 1997

2. Some Issues in Bottom-Up Capital Calculation

The standard formula for the solvency capital requirement adopts a bottom-up or modular approach.

Individual risks (sub-units) are transformed into capital charges $\mathsf{SCR}_1, \ldots, \mathsf{SCR}_d$. These are then combined to calculate the overall solvency capital requirement SCR. ([CEIOPS-06, 2006], page 71)

The combination operation may involve a calculation of the following kind:

$$\mathsf{SCR} = \sqrt{\sum_{i=1}^{d} \sum_{j=1}^{d} \rho_{ij} \mathsf{SCR}_i \mathsf{SCR}_j}$$

where the ρ_{ij} are the "correlations" between the risks. (for, example [CEIOPS-06, 2006], page 98)

Where is the Principle in This?

Suppose

- we measure risks with a quantile-based (Value-at-Risk) approach (SCR$_i = \mathrm{VaR}_\alpha(L_i)$, SRC $= \mathrm{VaR}_\alpha(L)$, $\alpha > 0.5$);

- the risks $(L_1, \ldots, L_d)$ are jointly normal with zero mean and correlations given by ρ_{ij}.

(More generally, we could consider any positive-homogeneous risk measure (such as cVaR/expected shortfall) in first assumption and any centred elliptical distribution (such as multivariate Student t) in second.)

Short Derivation of Aggregation Rule

$$\mathrm{sd}(L) = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathrm{sd}(L_i)\mathrm{sd}(L_j)}$$

Now $\mathrm{VaR}_\alpha(L) = \lambda_\alpha \mathrm{sd}(L)$ and $\mathrm{VaR}_\alpha(L_i) = \lambda_\alpha \mathrm{sd}(L_i)$ where λ_α is the α-quantile of standard normal. This yields

$$\mathrm{VaR}_\alpha(L) = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathrm{VaR}_\alpha(L_i)\mathrm{VaR}_\alpha(L_j)}$$

$$\mathsf{SCR} = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}\rho_{ij}\mathsf{SCR}_i\mathsf{SCR}_j} \quad .$$

Issues with this style of bottom-up

- It is only underpinned by theoretical principles in a very specific and unrealistic model of the risk universe.

- It is dependent on the widely misunderstood concept of correlation.

- The kinds of risks where we have reliable empirical experience of typical values are in the minority (e.g. financial market risks, and even then only at shorter time horizons)

- Can we trust "experts" to deliver correlations in other cases? There are consistency requirements: every ρ_{ij} should be compatible with the distribution of L_i and L_j. The matrix (ρ_{ij}) must be positive semi-definite. It is quite easy to specify nonsensical correlation matrices.

How to Account for Tail Dependence?

"Further analysis is required to assess whether linear correlation, together with a simplified form of tail correlation may be a suitable technique to aggregate capita requirements for different risks." [CEIOPS-06, 2006] (page 75)

"When selecting correlation coefficients, allowance should be made for tail correlation. To allow for this, the correlations used should be higher than simple analysis of relevant data would indicate." [CEIOPS-06, 2006] (page 142)

Is the sum of capital charges a bound for SCR?

Suppose again that risk capital charges have the quantile interpretation so that $\mathsf{SCR}_i = \mathrm{VaR}_\alpha(L_i)$ and $\mathsf{SCR} = \mathrm{VaR}_\alpha(L)$.

In the case where we have no diversification (comonotonic risks $L_i = u_i(Z)$, $i = 1, \ldots, d$) we can compute that

$$\mathsf{SCR} = \sum_{i=1}^{d} \mathsf{SCR}_i$$

Fallacy:

"this is an upper bound for the solvency capital requirement under all dependence assumptions for $(L_1, \ldots, L_d)$."

Superadditive Capital

Actually, it is possible to construct models for $(L_1, \ldots, L_d)$ with unusual dependence structures such that

$$\mathrm{VaR}_\alpha(L) > \sum_{i=1}^{d} \mathrm{VaR}_\alpha(L_i) = \mathsf{SCR}$$

It is also possible to find violations for independent risks when individual loss distributions are strongly skewed.

- To rectify this problem we would have to base risk measurement and capital charges on a subadditive risk measure (like expected shortfall).

- Many argue that the models leading to superadditivity are too implausible to consider, but they do undermine our principles!

Better Bottom-Up

- Copulas are a better theoretical tool for combining the individual capital charges. They avoid tricky consistency requirements imposed by working with linear correlations.

- Implicitly aggregation based on the Gauss copula has been used in insurance for years. For example @RISK by Palisade software implicitly uses the Gauss copula to perform Monte Carlo risk analysis.

- However, calibration remains a problem. Copula parameters are usually inferred from matrices of rank correlations, but are we expert enough to set these?

- Bottom-up approaches require the exogenous specification of parameters determining the dependence model.

3. Some Issues in Top-Down Capital Calculation

In a top-down approach the correlations are endogenous and result from specifying the mutual dependence of risks across the enterprise on common risk drivers or factors.

$$L_i = f_i(\text{common factors}, \text{idiodyncratic errors}), \quad i = 1, \ldots, d.$$

Generally these models are handled by Monte Carlo, i.e. the generation of scenarios for the common driving factors.

They appeal because they are structural and explanatory.

Advantages

- For an internal solvency capital model, this would be the more "principles-based" way to proceed.
- A natural framework for risk-based allocation of solvency capital to business units which opens door to risk-based performance measurement (RORAC).
- A framework for actual computation of the diversification benefit and attribution of that benefit to sub-units.
- Framework for sensitivity analyses with respect to common factors and model risk studies with respect to model assumptions.
- Tail correlation may be studied in terms of extreme outcomes in key risk drivers.

Capital Allocation

We require a method of breaking up the overall solvency capital requirement into a vector of capital allocations $(\mathrm{EC}_1, \ldots, \mathrm{EC}_d)$ such that

$$\mathsf{SCR} = \sum_{i=1}^{d} \mathrm{EC}_i$$

If we base our capital adequacy computation on a risk measure, such as VaR, it is known that a rational and fair way of doing this is Euler allocation [Tasche, 1999]. In the case of VaR we have $\mathsf{SRC} = \mathrm{VaR}_\alpha(L)$ and the capital allocations are given by

$$\mathrm{EC}_i = E\left(L_i \mid L = \mathrm{VaR}_\alpha(L)\right),$$

where EC_i is known as the VaR contribution of business unit i. Contributions can be estimated by Monte Carlo.

Diversification Scoring

Tasche [Tasche, 2006] defines diversification factors as follows:

$$\text{DF} = \frac{\text{SRC}}{\sum_{i=1}^{d} \text{SRC}_i}$$
$$\text{DF}_i = \frac{\mathrm{EC}_i}{\text{SRC}_i}$$

The former measures portfolio diversification - overall benefit in terms of reduction in solvency capital that the business units achieve by being together within the enterprise.

The latter measures effect of diversification for unit i - the benefit to business unit i in terms of reduction in solvency capital achieved by belonging to enterprise.

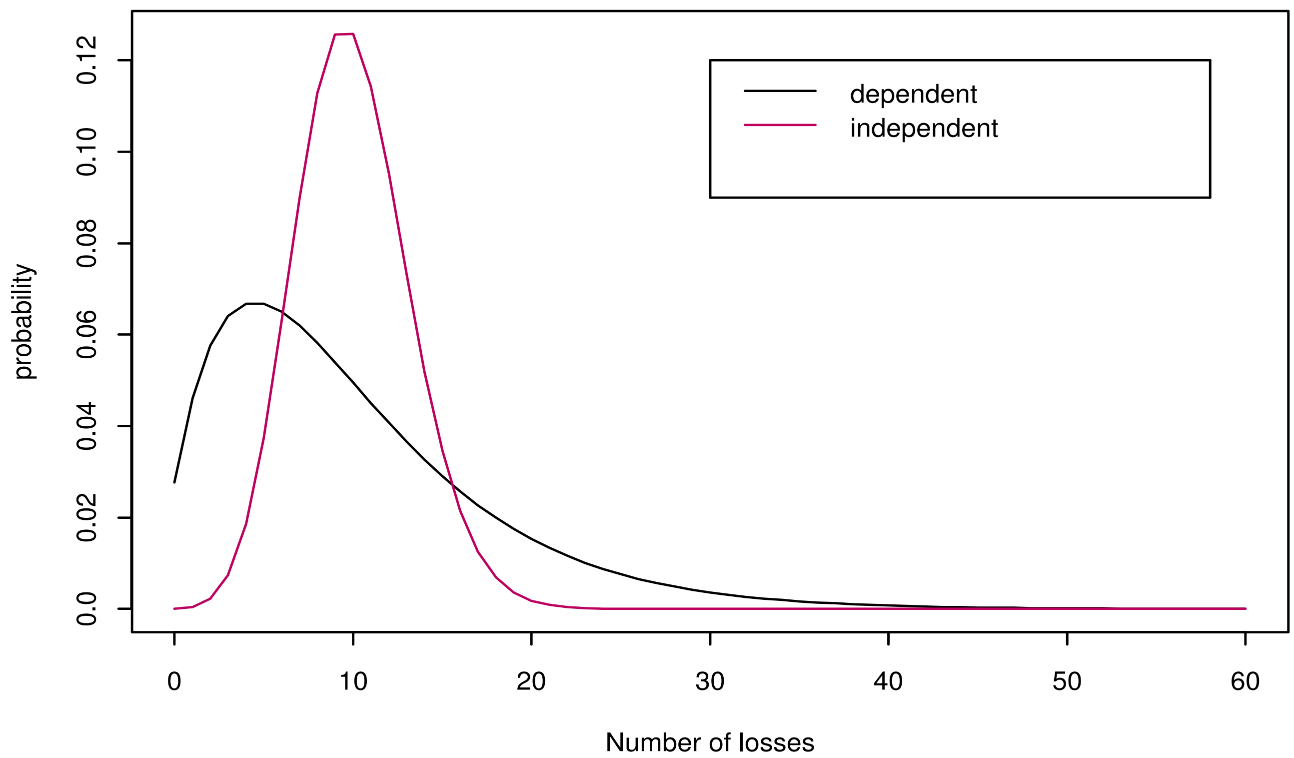
Issues

The conclusions about capital adequacy and risk-based performance comparison are only as good as the underlying models, which need to be built by skilled craftsmen. The biggest issue is the sensitivity of the results to the model inputs, in particular the model components specifying the dependence of risks on common factors.

Seemingly innocuous assumptions about correlations can have large effects.

Consider following example from credit risk. By adding a common factor that induces a default correlation of 0.005 between every pair of counterparties, we inflate tail of loss distribution.

Impact of Dependence on Credit Loss Distribution



Comparison of the loss distribution of a homogeneous portfolio of 1000 loans with a default probability of 1% assuming (i) independent defaults and (ii) a default correlation of 0.5%.

References

[CEIOPS-06, 2006] CEIOPS-06 (2006). *Draft advice to the European Commission in the Framework of the Solvency II project on Pillar I issues - further advice*. CEIOPS (Committee of European Insurance and Occupational Pensions Supervisors).

[Kuritzkes et al., 2002] Kuritzkes, A., Schuermann, T., and Weiner, S. (2002). Risk measurement, risk management and capital adequacy in financil conglomerates. Technical Report 3, Wharton Financial Institutions Centre.

[McNeil et al., 2005] McNeil, A., Frey, R., and Embrechts, P. (2005). *Quantitative Risk Management: Concepts, Techniques and Tools*. Princeton University Press, Princeton.

[Tasche, 1999] Tasche, D. (1999). Risk contributions and performance measurement. Preprint, TU-Munich.

[Tasche, 2006] Tasche, D. (2006). measuring sectoral diversification in an asymptotic multifactor framework. *Journal of Credit Risk*, 2(3):33–55.

[Tiner, 2006] Tiner, J. (2006). Speech by John Tiner, Chief Executive, FSA at ABI conference on Solvency II and IASB Phase II, 6 April 2006.

[Treasury and FSA, 2006] Treasury and FSA (2006). *Solvency II: a new framework for prudential regulation of insurance in the EU*. HM Treasury and FSA. Discussion paper.