

Report on the workshop on

Lie algebras, vertex algebras and automorphic forms

at the ICMS, 31 August - 4 September 2009

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1 Relation to the original proposal

There were no significant deviations from the original proposal.

2 Overview

In 1979 Conway and Norton made a precise conjecture about a relation between the largest sporadic finite simple group and the theory of modular forms. This was confirmed 13 years later by Borcherds. In order to prove the conjecture he introduced new algebraic structures, namely generalized Kac-Moody algebras and vertex algebras. Shortly after he found that generalized Kac-Moody algebras are sometimes connected with automorphic forms on orthogonal groups. Since then there has been a lot of progress on these subjects and their relations. However some of the main problems are still open, e.g. the generalized moonshine conjecture and the classification problem for generalized Kac-Moody algebras and vertex algebras. These questions were discussed at the workshop.

3 Report

Borcherds' proof of Conway and Norton's moonshine conjecture in 1992 showed that vertex algebras, generalized Kac-Moody algebras and automorphic forms are closely related. However several important questions are still open. Among these are:

- Generalized Moonshine

- Moonshine for other groups
- Classification of vertex algebras and generalized Kac-Moody algebras
- Construction of vertex algebras and generalized Kac-Moody algebras
- Automorphic forms related to vertex algebras and generalized Kac-Moody algebras
- Related subjects as hyperbolic reflection groups and moduli spaces

The workshop was devoted to these topics.

The organizers gave three overview talks which defined the background for the other results that were presented and introduced the younger scientists to the subject.

Generalized Kac-Moody algebras appear as cohomology groups of certain vertex algebras. In these cases their denominator identities conjecturally are automorphic forms on orthogonal groups, more precisely automorphic products. This was described in Höhn's talk.

Scheithauer gave a short introduction into automorphic forms on orthogonal groups and reviewed the classification of generalized Kac-Moody algebras whose denominator identity is a reflective automorphic product of singular weight.

The classification of generalized Kac-Moody algebras with automorphic denominator identity is closely related to the classification of hyperbolic reflection groups. Nikulin described recent results on the finiteness and classification of arithmetic hyperbolic reflection groups.

In a public lecture Kac introduced Poisson vertex algebras and vertex algebras and explained their relation to classical field theory and quantum field theory.

Several new important results were presented at the workshop.

Norton extended the original moonshine conjecture in the following way: For each element g in the monster there is a module V_g on which a central extension $\hat{C}(g)$ of the centralizer of g in the monster acts so that the traces $T_g(h, \tau) = q^{-1} \sum_{n \in \mathbb{Z}} \text{tr } h|_{(V_g)_n} q^n$ are modular functions for genus 0 groups for all $h \in \hat{C}(g)$. Carnahan essentially solved this problem in his Ph.D. thesis. In his talk he described the construction of the modules V_g as vertex algebras.

The moonshine vertex operator algebra is conjectured to be the unique vertex operator algebra V which has central charge 24, only one irreducible module and $V_1 = 0$. Lam presented progress made on this question using Virasoro frames.

Dong explained a new characterization result for the vertex operator algebra $L(1/2, 0) \otimes L(1/2, 0)$.

Yamauchi described the construction of n -transposition groups as automorphism groups of vertex operator algebras. For the vertex operator algebras

related to the baby monster and the largest Fischer group it gives 4- and 3-transposition groups. The construction also explains a relation between these groups and the affine E_6 - and E_7 -diagram observed by McKay.

Griess presented further results on automorphism groups of vertex operator algebras related to sublattices of the Leech lattice and the root lattice E_8 .

One of the main open problems in the theory of vertex operator algebras is the rationality of orbifold vertex operator algebras. Miyamoto described a new result on the structure of the tensor category associated to a C_2 -cofinite vertex operator algebra. This implies rationality in certain cases.

Mason studied logarithmic vertex operator algebras, i.e. C_2 -cofinite vertex operator algebras with only finitely many modules. He formulated the conjecture that the trace functions associated to logarithmic vertex operator algebras are quasimodular forms.

Milas described new properties of the triplet vertex algebra $\mathcal{W}(p)$ and the vertex algebra $\mathcal{W}_{2,p}$.

Schweigert constructed a classifying algebra for the boundary conditions of a rational conformal field theory using the mapping class group.

McKay reviewed some moonshine phenomena which have not found an explanation yet.

Gritsenko derived classification results for reflective automorphic forms which can be obtained as restrictions of Borcherds' function Φ_{12} , the denominator identity of the fake monster algebra.

The denominator identity of a certain generalized Kac-Moody superalgebra is an automorphic form on $O_{10,2}(\mathbb{R})$. Using this function Kondo constructed a model of the 10-dimensional moduli of Enriques surfaces in 186-dimensional projective space.

Vector valued modular forms can be used to construct automorphic forms on orthogonal groups and appear as characters of rational conformal field theories. Therefore they play a fundamental role in these theories. Bantay and Gannon presented a theory of vector valued modular forms and described some applications.

Skoruppa showed that Jacobi forms of critical weight can be identified with invariants of suitable Weil representations. This can be used to study orthogonal and Siegel modular forms of critical weight.

Cummins classified torsion free, genus one congruence subgroups.

Since the work of Zhu it is well known that the genus one partition function of a vertex operator algebra has modular properties. Tuite defined genus two partition functions for vertex operator algebras and superalgebras. He calculated these functions explicitly for Heisenberg and lattice vertex operator algebras and for fermionic orbifolds and showed that they are automorphic forms on suitable groups.

The modular properties of the McKay-Thompson series are plausible in the context of representation theory of vertex algebras. However the genus zero property is still mysterious. Duncan introduced certain Rademacher sums for subgroups Γ of $SL_2(\mathbb{Z})$ and showed that they have modular properties if Γ has

genus zero and coincide with the corresponding hauptmodul in that case.

Ray developed a theory of Cartan subalgebras for generalized Kac-Moody algebras and locally finite Lie algebras.

Kumar explained his proof of the Cachazo-Douglas-Seiberg-Witten conjecture for symmetric spaces.

Scharlau gave an overview on the classification of integral hyperbolic lattices whose reflection group has finite index in the automorphism group.

By results of Nikulin and Vinberg the number of these lattices is finite. In his talk Allcock announced a complete classification in signature $(2, 1)$.

Basak reported on his work relating abstract Coxeter groups, complex hyperbolic reflection groups and the bimonster.

Dijkgraaf, Verlinde and Verlinde showed that the generating function for dyon counting in $N = 4, d = 4$ string theory is given by the inverse of the square of the denominator identity of a certain generalized Kac-Moody superalgebra. This formula was used to reproduce the macroscopic Bekenstein-Hawking entropy. The physical meaning of the generalized Kac-Moody superalgebra is still unclear. Cheng showed the reflection hyperplanes of the Weyl group correspond to the walls of marginal stability so that the Weyl group can be interpreted as the group of wall-crossing. In this way at least part of the symmetry has a physical interpretation.

The workshop had 43 participants from 9 countries, among them the leading experts in the different fields and 11 younger researchers. The above description of the talks shows that a number of excellent new results was presented. Also many new ideas emerged during the discussions (see for example Miyamoto's preprint arXiv:0909.3665) and several collaborations were initiated. In summary the workshop was a great success.

We thank the ICMS for making this workshop possible and for its excellent support.