Operator algebras attached to affine transformations on adele spaces

joint work with J. Cuntz

Xin Li

University of Muenster

Our main result

- K: a global field
- $K \rtimes K^\times$: the semidirect product of $(K, +)$ and $(K^\times, \cdot)$ with respect to the multiplicative action

Our main result

- K: a global field
- $K \rtimes K^\times$: the semidirect product of $(K, +)$ and $(K^\times, \cdot)$ with respect to the multiplicative action
- $K \rtimes K^\times$ acts on $\mathbb{A}_\infty$ via affine transformations
  $\rightsquigarrow$ operator algebra $C_0(\mathbb{A}_\infty) \rtimes (K \rtimes K^\times)$

Our main result

- K: a global field
- $K \rtimes K^\times$: the semidirect product of $(K, +)$ and $(K^\times, \cdot)$ with respect to the multiplicative action
- $K \rtimes K^\times$ acts on $\mathbb{A}_\infty$ via affine transformations
  $\rightsquigarrow$ operator algebra $C_0(\mathbb{A}_\infty) \rtimes (K \rtimes K^\times)$

idea: enlarge $C_0(\mathbb{A}_\infty)$ so that the action of $K \rtimes K^\times$ becomes inner
($C_0(\mathbb{A}_\infty)$ is the algebra of continuous functions $\mathbb{A}_\infty \to \mathbb{C}$ which are small outside compact sets)

Our main result

- K: a global field
- $K \rtimes K^\times$: the semidirect product of $(K, +)$ and $(K^\times, \cdot)$ with respect to the multiplicative action
- $K \rtimes K^\times$ acts on $\mathbb{A}_\infty$ via affine transformations
  $\rightsquigarrow$ operator algebra $C_0(\mathbb{A}_\infty) \rtimes (K \rtimes K^\times)$

  idea: enlarge $C_0(\mathbb{A}_\infty)$ so that the action of $K \rtimes K^\times$ becomes inner
  ($C_0(\mathbb{A}_\infty)$ is the algebra of continuous functions $\mathbb{A}_\infty \to \mathbb{C}$ which are small outside compact sets)
- analogously: $K \rtimes K^\times \curvearrowright \mathbb{A}_f \rightsquigarrow C_0(\mathbb{A}_f) \rtimes (K \rtimes K^\times)$

Our main result

- K: a global field
- $K \rtimes K^\times$: the semidirect product of $(K, +)$ and $(K^\times, \cdot)$ with respect to the multiplicative action
- $K \rtimes K^\times$ acts on $\mathbb{A}_\infty$ via affine transformations
  $\rightsquigarrow$ operator algebra $C_0(\mathbb{A}_\infty) \rtimes (K \rtimes K^\times)$

  idea: enlarge $C_0(\mathbb{A}_\infty)$ so that the action of $K \rtimes K^\times$ becomes inner
  ($C_0(\mathbb{A}_\infty)$ is the algebra of continuous functions $\mathbb{A}_\infty \to \mathbb{C}$ which are small outside compact sets)
- analogously: $K \rtimes K^\times \curvearrowright \mathbb{A}_f \rightsquigarrow C_0(\mathbb{A}_f) \rtimes (K \rtimes K^\times)$

Theorem

For every global field K, we have

$$C_0(\mathbb{A}_\infty) \rtimes (K \rtimes K^\times) \cong C_0(\mathbb{A}_f) \rtimes (K \rtimes K^\times).$$