Metric number theory and Diophantine equations

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EU-Young and Mobile Workshop
Dynamical Systems and Number Theory
May 17, 2010 - May 19, 2010 .

In 1909 Borel proved that almost all numbers are "normal numbers".

In other words: the system $\left(\langle 2^k x\rangle\right)_{k\geq 1}$ is uniformly distributed (mod 1) for almost all real x.

This can be seen as a consequence of the (pointwise) ergodic theorem for the transformation

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What happens if the sequence $(2^k)_{k \geq 1}$ is replaced by an other "fast" growing sequence $(n_k)_{k \geq 1}$ of positive integers?

It turns out that systems of the form $(\langle n_k x \rangle)_{k \geq 1}$ or $(f(n_k x))_{k \geq 1}$ resemble many properties of systems of i.i.d. random variables.

The precise behaviour of these systems is intimately related to the number of solutions (k, l) of Diophantine equations of the form

$$a n_k \pm b n_l = c, \qquad a, b, c \in \mathbb{Z}.$$

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A typical result:

Theorem (A., Berkes, 2010)

Let f be a "nice", 1-periodic function. Assume that the number of solutions (k, l) of the Diophantine equation

$$an_k - bn_k = c$$

is "not too large". Then the system $(f(n_k x))_{k \geq 1}$ satisfies the CLT and LIL in exactly the same way as a sequence of i.i.d. random variables.