Bernstein-Durrmeyer Operators with General Weight Functions

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Let ρ be a non-negative bounded (regular) Borel measure on the simplex

$$\mathbb{S}^d := \{x = (x_1, \ldots, x_d) \in \mathbb{R}^d \ : \ 0 \le x_1, \ldots, x_d \le 1, \ x_1 + \cdots + x_d \le 1\}.$$

We assume that $\operatorname{supp}(\rho) \setminus \partial\mathbb{S}^d \neq \emptyset$. The Bernstein basis polynomials of degree $n \in \mathbb{N}$ are defined by

$$B_\alpha(x) := \frac{n!}{\alpha_0! \alpha_1! \cdots \alpha_d!} (1 - x_1 - \cdots - x_d)^{\alpha_0} x_1^{\alpha_1} \cdots x_d^{\alpha_d},$$

where $\alpha_0, \alpha_1, \ldots, \alpha_d \in \mathbb{N} \cup \{0\}$ and $\alpha_0 + \alpha_1 + \cdots + \alpha_d = n$. We introduce the Bernstein-Durrmeyer operator with weight ρ

$$\mathbf{M}_{n,\rho} f := \sum_{\alpha_0 + \cdots + \alpha_d = n} \frac{\int_{\mathbb{S}^d} f \, B_\alpha \, d\rho}{\int_{\mathbb{S}^d} B_\alpha \, d\rho} B_\alpha$$

for $f \in L^q_\rho(\mathbb{S}^d)$, $1 \le q < \infty$, or $f \in C(\mathbb{S}^d)$. The operator $\mathbf{M}_{n,\rho}$ is linear and positive, and it reproduces constant functions. It generalizes the well-known Bernstein-Durrmeyer operators with Jacobi weights. A motivation for this generalization comes from learning theory.

We discuss properties of the operator $\mathbf{M}_{n,\rho}$ such as spectral properties or the structure of the associated reproducing kernel Hilbert space. We make first steps in understanding convergence of the operators. In particular, we give necessary and sufficient conditions on the measure ρ such that uniform convergence holds for all continuous functions on $\mathbb{S}^d$.

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