

Report on ICMS Workshop

“Multivariate Approximation and Interpolation with Applications”

Edinburgh, September 6–10, 2010

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1 Deviations from the original proposal

A number of colleagues could not come (Professors Cohen, Costantini, Gasca, Kunoth, Levesley, Mason, Mazure, Neamtu, Pinkus, Wendland). However, their place was taken by other well established researchers, such as Prof Carnicer, Prof Conti, Prof Beatson, and a larger than originally planned number of younger participants, including Ms Andriamaro, Dr Babenko, Dr Berdysheva, Mr Chernih, Dr Hangelbroek, Dr Hormann, Mr Rabarison, Dr Reimers, Dr Sampoli, Dr Speleers, Mr Yeo.

2 Short description of the meeting

Approximation theory has evolved from classical work by Chebyshev, Weierstrass and Bernstein into an area that combines a deep theoretical analysis of approximation with insights leading to the invention of new computational techniques. Such invaluable tools of modern computation as orthogonal polynomials, splines, finite elements, Bézier curves, NURBS, radial basis functions, wavelets and subdivision surfaces have been developed and analysed with the prominent help of ideas coming from approximation theory. The workshop was devoted to the approximation of functions of two or more variables. This area has many challenging open questions and its wide variety of applications includes problems of computer aided design, mathematical modelling, data interpolation and fitting, signal analysis and image processing. The focus was on the following research topics: approximation and interpolation with multivariate polynomials, splines, and radial basis functions, linear and non-linear subdivision, adaptive and multiresolution methods of approximation, shape preserving approximation.

3 Report

The workshop gathered many leading experts in Approximation Theory and a significant number of promising early career researchers and enabled them to follow the latest developments, exchange new ideas and establish new and support existing research contacts. The workshop had a truly international character, with its 46 participants coming from 14 countries (Australia, Austria, Belgium, Canada, France, Germany, Israel, Italy, New Zealand, Norway, Spain, Switzerland, USA, UK).

The main research topics of high current interest in multivariate approximation were well represented by excellent talks.

- The questions of approximation and interpolation with polynomials and piecewise polynomial splines were addressed by Yuliya Babenko, Elena Berdysheva, Kai Hormann, Gunter Nrnberger, Ulrich Reif, Paul Sablonnire, Larry Schumaker, Hendrik Speleers.
- State of the art research on radial basis functions was presented by Rick Beatson, Thomas Hangelbroek, Kerstin Hesse, Amos Ron, Robert Schaback, Milvia Rossini, Joe Ward.

- The talks in the area of subdivision methods were given by Costanza Conti, Nira Dyn, Michael Floater, Tomas Sauer, Joachim Stöckler, Johannes Wallner.
- Adaptive and multiresolution methods of approximation were covered by Peter Binev, Shai Dekel, Bin Han.
- Shape preserving approximation was addressed by Jess Carnicer, Carla Manni, Juan Manuel Pea, Malcolm Sabin, Maria Lucia Sampoli.

Many of the talks had a strong applied or interdisciplinary component, in particular those by Rick Beatson, Kerstin Hesse, Milvia Rossini (scattered data fitting), Brad Baxter (financial mathematics), Piter Binev, Carla Manni (numerical PDE), Jess Carnicer, Juan Manuel Pea, Malcolm Sabin, Maria Lucia Sampoli (geometric modelling).

With so many good talks, it is difficult to select highlights, but the talks by Larry Schumaker on multivariate splines, by Robert Schaback and Joe Ward on radial basis functions, and by Costanza Conti on subdivision are definitely worth mentioning.

Prof Conti presented new algebraic conditions that fully identify the exponential polynomials reproduction properties of a given convergent, univariate, binary, non-stationary subdivision scheme, which allow creation of new non-stationary subdivision schemes with enhanced reproduction properties important for shape preservation.

Prof Schaback gave a comprehensive survey of the bases of spaces spanned by translates of symmetric positive definite kernels, emphasising their stability properties and generalized Lebesgue constants. Note that the kernel techniques are the basis of several very promising meshless methods for the solution of partial differential equations. Their investigation by the methods of approximation theory is necessary for the understanding of these methods.

Prof Schumaker, in his talk on spline spaces on TR-meshes with hanging vertices, presented difficult new results on the dimension of these spaces, which are often used in finite element analysis, but have not been studied in approximation theory before.

Prof Ward presented a new paradigm for error estimates with radial basis functions and spherical basis functions. In contrast to the traditional reproducing kernel Hilbert spaces approach, the new methods allow bounds in terms of Sobolev and Besov norm of the function, which greatly extends the applicability of the bounds.

Several very interesting talks addressed emerging research areas, in particular Prof Sabin's talk on the notion of geometric precision, Prof Manni's talk on isogeometric analysis and Dr Baxter's talk on a surprising link between approximation theory and the exponential functionals of Brownian motion.

In their responses to the online questionnaires after the workshop, participants highlight the important role of the workshop in maintaining their existing and helping initiate new research collaborations.

A comprehensive web page at the ICMS site has been developed with the kind help by Ms Johanna Kytöharju, containing information for participants and scientific community. It shows a final list of participants and schedule, as well as the abstracts of the presentations.

The workshop venue was very good and well-suited, the facilities were excellent and ICMS organisation perfect and very much appreciated.

Social events (a visit to Rosslyn Chapel on Wednesday afternoon and the workshop dinner at the Magnum restaurant) were well attended and enjoyed.

The Journal of Computational and Applied Mathematics has approved our proposal to publish a special issue devoted to the proceedings of the workshop. The deadline for the papers is 15 January 2011, and the volume edited by the two organizers of the workshop will be ready for publication by mid 2011.

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