Resonances of complex dynamics

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Abstracts

Astorg, Matthieu

Wandering domains in higher dimension

A wandering domain for a holomorphic map f is a Fatou component that is not eventually periodic. A famous theorem due to Sullivan asserts that rational maps in one variable do not have wandering domains. On the other hand, there are numerous examples of transcendental entire maps with wandering domains. Until recently, it was an open question whether a rational or polynomial map in higher dimension could have them. The answer is yes; we will present a construction of a polynomial map in two complex variables with that property. This is a joint work with X. Buff, R. Dujardin, H. Peters and J. Raissy

Barański, Krzysztof

Pressure and conformal measures for transcendental meromorphic maps

I will talk on some elements of thermodynamic formalism which hold for a large class of transcendental meromorphic maps. I will show the existence of the topological pressure, Bowen's formula determining the hyperbolic dimension of the Julia set and conditions for the existence of conformal measures for arbitrary meromorphic maps from class S with a logarithmic tract and all maps from class B with a logarithmic tract and a 'minimal hyperbolicity' condition. This is joint work with Bogusława Karpińska and Anna Zdunik

Benini, Anna Miriam

How many repelling orbits can be rationally invisible?

Let f be a transcendental entire functions with finitely many singular values, for which periodic rays exist and land. A repelling periodic point is rationally invisible if it is not the landing point of any periodic ray. We show that the number of periodic orbits whose points are rationally invisible is bounded by the number of singular values of f. This is joint work with Nuria Fagella.

Bonk, Mario

Sullivan's dictionary, Cannon's conjecture, and expanding Thurston maps

There are many analogies between the iteration theory of rational functions and the theory of Kleinian groups. This is often referred to as Sullivan's dictionary, because he pointed out some of these analogies explicitly. In my talk I will discuss some aspects of Sullivan's dictionary related to Cannon's conjecture in geometric group theory and the geometry of fractals generated by the dynamics of expanding Thurston maps.

Chéritat, Arnaud

Similarities between Julia sets of exponential maps and hedgehogs of quadratic maps, definition of a toy model

We will motivate the definition of a toy model introduced by Buff and Chéritat 10 years ago. Its purpose is to give an idea of what hedgehogs of quadratic polynomials should look like. We believe the model to be faithful at least topologically and maybe a little bit more.

Devaney, Bob

Dynamics of the maps $z^n + \frac{C}{z^n}$ *; why* $n = 2$ *is crazy*

In this talk we show that there are three major reasons why the family of maps $z^n + \frac{C}{z^n}$ is very different from the family $z^n + \frac{C}{z^n}$. One reason is there is no "McMullen domain" when $n = 2$. A second is that there are infinitely many "Mandelpinski necklaces" around the McMullen domain when $n > 2$. And a third is that the Julia sets converge to the unit disk when C tends to zero in the case $n = 2$, but this does not occur if $n > 2$.

Dobbs, Neil

Birkhoff averages of perturbations.

Birkhoff averages (of an observable along orbits) are objects of interest when investigating statistical behaviour of a dynamical system. If there is a unique physical measure, the Birkhoff averages will converge, for almost every orbit, to the space average (i.e. the integral) of the observable, so the physical measure captures important statistical properties of the dynamical system. However, in the quadratic family, for example, physical measures don't always exist, and even when they do, they don't necessarily depend continuously on the parameter. In joint work with Alexey Korepanov, we examine what happens for finite time Birkhoff averages for nearby parameters.

Drasin, David

The sharpness of the Mues conjecture

In 2013, K. Yamanoi (Proc London Math. Soc.) confirmed the full A. A. Goldberg conjecture. A special case had been formulated earlier by E. Mues, and this talk focuses on that less complicated situation. If f is a transcendental meromorphic function, the Mues conjecture asserts that $\sum_{a\neq\infty} \delta(a, f') \leq 1$, where we use the standard notation of the Nevanlinna theory of meromorphic functions. In this talk, we refine the method of quasiconformal modification to show that the Mues conjecture is best possible. Note that the conjecture is trivial for entire functions, since Nevanlinna's classical relation $\sum_a \delta(a, f) \leq 2$ is sharp. The authors acknowledge the influence of W. Bergweiler and A. Eremenko for this project. This is joint work with Jingjing Qu

Eremenko, Alexandre

Interaction between complex dynamics and function theory

Most works in holomorphic dynamics use some general function theory tools, including advanced tools like Ahlfors theory, minimum modulus theorems etc. But there are several higher levels of interaction between these two subjects: a) A question of dynamics can stimulate a further development in "pure" function theory, and most interestingly, b) a new result of pure function theory can be proved using dynamical considerations. As a further development of b), it even happens that new questions in dynamics arise from potential applications of dynamics to the general theory of functions. All these patterns will be discussed and illustrated with specific examples from the work of Walter Bergweiler.

Fagella, Nuria

Fatou components of transcendental maps and singularities

We explore the relationship between periodic and non-periodic Fatou components of transcendental meromorphic functions and singularities of the inverse function.

Geyer, Lukas

Dynamical moduli spaces and multipliers of periodic points

One of the fundamental questions in complex dynamics is to understand the structure of and coordinates on *moduli space* $\mathcal{M}_d$, i.e., the space of Mobius conjugacy classes of rational maps of some given degree d. Multipliers at fixed points and periodic points are conjugacy invariants, so a

natural question is how uniquely they determine conjugacy classes. For square degrees $d = n^2$ there is always a one dimensional subvariety $\Lambda_d \subset \mathcal{M}_d$ consisting of rational maps which are semi-conjugate to multiplication by n on a torus, with all multipliers constant, the 'flexible Lattés locus'. However, McMullen showed that for every degree d there exist numbers N_d and such that the multipliers of periodic points up to period N_d determine at most K_d conjugacy classes in $\mathcal{M}_d \setminus \Lambda_d$.

Although not much is known about N_d and K_d for $d \geq 3$, we will find "generic" versions of these, i.e., $N_d'$and K_d' such that multipliers up to period N_d' determine at most K_d' conjugacy classes in an open dense subset of $\mathcal{M}_d$.

We also consider the same problem in the moduli space $\mathcal{M}_d^P$ of polynomials instead of rational maps, and show that in this case, *fixed point data* (multiplicity and holomorphic indices of all fixed points, equivalent to fixed point multipliers for polynomials without multiple fixed points) determines at most $(d-2)!$ conjugacy classes in $\mathcal{M}_d^P$ for $d \geq 2$.

This is joint work with Adam Epstein (University of Warwick, UK).

Kisaka, Masashi
Construction of transcendental entire functions of arbitrarily slow growth with prescribed polynomial dynamics
We show that for a given polynomial P with deg $P > 1$, there exists a transcendental entire function with arbitrarily slow growth which has the dynamics of P as its subdynamics. This is done by using a technique used by Bergweiler and Eremenko. We also show some applications of this result.

Karpińska, Bogusława
Escaping points in the boundaries of Baker domains
This talk concerns the dynamical behaviour of points in the boundaries of simply connected invariant Baker domains U of meromorphic maps with finite degree on U. We show that the set of boundary points that escape to infinity under iteration can have zero or full harmonic measure, depending on the type of Baker domain. Additionally we present some extensions to the infinite degree case. The talk is based on a joint work with Krzysztof Baranski, Nuria Fagella and Xavier Jarque.

Lyubich, Mikhail
On the local connectivity and the area problems in the quadratic dynamics
We will discuss several closely related problems: on the local local connectivity of the Mandelbrot set (MLC) and on the local connectivity and area of Julia sets for quadratic polynomials. Specifically we will focus on "infinitely renormalizable parameters of bounded satellite type" (all the notions will be explained in the talk). We will show how to construct some parameters of this type where MLC holds and whose Julia sets are locally connected and have positive area. It requires some understanding of a one-parameter transcendental family produced as the scaling limit of the quadratic family near the golden Siegel parameter. Based on a joint work with Dima Dudko.

Meyer, Daniel
Quasispheres and Expanding Thurston maps
A quasisymmetric map is one that changes angles in a controlled way. As such they are generalizations of conformal maps and appear naturally in many areas, including complex analysis and geometric group theory. A quasisphere is a metric sphere that is quasisymmetrically equivalent to the standard 2-sphere. An important open question is to give a characterization of quasispheres. This is closely related to Cannon's conjecture. This conjecture may be formulated as stipulating that a group that "behaves topologically'" as a Kleinian group "is geometrically" such a group. Equivalently, it stipulates that the "boundary at infinity" of such groups is a quasisphere. A Thurston

map is a map that behaves "topologically" as a rational map, i.e., a branched covering of the 2-sphere that is postcritically finite. A question that is analog to Cannon's conjecture is whether a Thurston map "is" a rational map. This is answered by Thurston's classification of rational maps. For Thurston maps that are expanding in a suitable sense, we may define "visual metrics". The map then is (topologically conjugate) to a rational map if and only if the sphere equipped with such a metric is a quasisphere. This talk is based on joint work with Mario Bonk.

Nicks, Daniel
The quasi-Fatou set in quasiregular dynamics
We define the Julia set of a quasiregular map as the set of points satisfying a blowing-up property. The quasi-Fatou set is then the complement of this Julia set – in particular, there is no normality assumption on this quasi-Fatou set. Nonetheless, we will discuss some analogies between quasi-Fatou components of quasiregular maps and Fatou components of transcendental entire functions.

Ng, Patrick
Smale's mean value conjecture and related problems
In this talk, we will explain how the theory of amoeba and geometric function theory can be used the study of Smale's mean value conjecture and other related problems originated from Smale's work on the efficiency of Newton's method

Rippon, Phil
Multiply connected wandering domains of entire functions
Noel Baker showed that multiply connected Fatou components of transcendental entire functions must be wandering domains and have the geometric structure of nested annuli escaping to infinity. We describe joint work with Walter Bergweiler and Gwyneth Stallard on such multiply connected wandering domains, in which we gave a detailed analysis of the dynamical behaviour within them. We then describe joint work with Anna Benini and Gwyneth Stallard which uses the understanding of this dynamical behaviour to make progress on the old problem of when two entire functions that commute have identical Julia sets, and also briefly discuss ideas arising from this work. Techniques used from complex analysis include contraction of the hyperbolic metric, many properties of harmonic functions, and Jensen's theorem.

Roesch, Pascale
From polynomials to rational maps some examples
We will discuss some influences and differences between polynomial dynamics and rational maps dynamics through examples such as Newton maps and Mc Mullen maps.

Shishikura, Mitsuhiro
Oscillating wandering domain for a transcendental entire function of class B
Bishop has introduced a technique called quasiconformal folding, and as an application, constructed a transcendental entire function of class B (with bounded singular values) having an oscillating wandering domain. However there seem to be several issues in the proof, so we propose a new construction which uses quasiconformal mappings, but not Bishop's quasiconformal folding. With our method, we can show that the obtained function has finite order. In the proof, we use an estimate on the variation of cross-ratio of 4 points on the sphere under quasiconformal mappings, which also lead to new proofs of theorems on conformality at a point and on parametric differentiability. This is a joint work with David Marti Pete.

Sixsmith, David
On simply-connected domains with connected preimage
We consider the following straightforward question: does there exist a transcendental entire function f, and two disjoint simply-connected domains U and V, such that $f^{-1}(U)$ and $f^{-1}(V)$ are both connected? The answer is surprising, and contrary to a series of published proofs, dating back to 1970. We answer this question, as well as discussing the error in these proofs.

Urbański, Mariusz
Thermodynamic formalism and iterated function systems in transcendental dynamics
I will survey the methods and results of thermodynamic formalism and conformal iterated function systems in the realm of iteration of meromorphic and entire transcendental functions. The primary applications of the former capture stochastic laws such as exponential decay of correlations, the central limit theorem and the law of iterated logarithm, and also provide exact formulas (of Bowen's type) for Hausdorff dimension of Julia sets and radial Julia sets. The latter, i.e. conformal iterated function systems are frequently used to get numerical estimates of the Hausdorff dimension of Julia sets and their other significant subsets.

van Strien, Sebastian
Complex methods in real dynamics
To follow

Wang, Yuefei
Dynamics on the one dimensional Berkovich space
We will talk about the dynamics of holomorphic maps over non-Archimedean fields, in comparison with that over the complex field.

Zdunik, Anna
Random non-hyperbolic exponential maps
I will present recent results obtained in a collaboration with Mariusz Urbański. For a random iteration of non-hyperbolic exponential maps $z \mapsto \lambda e^z$ (within some range of parameters) we prove the existence of random conformal and invariant measures supported on the radial Julia set, and describe the behaviour of the typical trajectory (in the sense of Lebesgue measure).