Nonlinear analysis and the physical and biological sciences (in honour of Jack Carr)

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International Centre for Mathematical Sciences, Edinburgh

Abstracts

Atiyah, Michael
The role of topology in non-linear analysis
In a wide range of non-linear problems it has become evident (but not obvious) that the algebraic topology of the function space can play a key role. Finite-dimensional submanifolds can give good approximations to stable phenomena. I will argue that Jack Carr's work on Centre Manifolds is of this type.

Ball, John
Jack Carr the mathematician
The talk will describe a selection of highlights from the mathematical work of Jack Carr.

da Costa, Fernando M
Sub-monolayer deposition models: similarity profiles and convergence rates
We present an infinite system of ODEs that is used in modelling the deposition of adatoms in a monolayer in a crystal facet and we will study the convergence of its solutions to similarity profiles, including their convergence rates. The techniques used consist in the decoupling of the infinite system into a two dimensional ODE and a linear triangular infinite ODE system; in the study of convergence rates of the two dimensional system using centre manifold tools, and in the analysis of the behaviour of the infinite system by a detailed analysis of the terms obtained by the application of the variation of constants formula. (This is part of a joint work with Joao Teixeira Pinto and Rafael Sasportes)

Dafermos, Constantine
Long time behaviour of solutions to scalar conservation laws
For scalar conservation laws in several space variables, with minimal restrictions on nonlinearity, the lecture will describe how to derive the long time behaviour of specially periodic solutions, with the help of basic ideas from topological dynamics and integral geometry.

Duncan, Dugald
Metastable patterns in solutions of a non-local equation
We consider the slow movement of layers in the solution of the nonlocal partial integro-differential equation $u_t = f(Lu) - u$, where L is a local space averaging operator and function f is sigmoidal with $f(0) = 0,\ f(0.5) = 0.5,\ f(1) = 1$. The equation was proposed by Bloomfield, Painter, Sherratt and Landini (2010) as a model of cell renewal in mosaic tissue with two different cell types, which together occupy the full volume. We need only model the fraction $0 \le u \le 1$ occupied by one cell type, and the process is driven by renewal according to the majority cell type in the surroundings, measured by the local average Lu.

We concentrate on the analysis of the problem in one space dimension. This has parallels with Jack and Bob Pego's 1989 paper on *Metastable patterns in solutions of* $u_t = \varepsilon^2 u_{xx} - f(u)$, in that a set of ODEs is derived which model the motion of the layer positions and that motion can be extremely slow.

This project was carried out with Jack Carr and our joint PhD student Ali Alshomrani.

Esteban, Maria J

Analytical and numerical results about symmetry and symmetry breaking for Caffarelli-Kohn-Nirenberg inequalities

Recent optimal results about the symmetry properties of optimizers for a class of Caffarelli-Kohn-Nirenberg inequalities will be presented in this talk.

Numerical computations show how symmetry breaking is linked, or not, to instability of symmetric optimizers. (This work has been done in collaboration with J. Dolbeault and M. Loss)

Fonseca, Irene

Mathematical analysis of novel advanced materials

Quantum dots are man-made nanocrystals of semiconducting materials. Their formation and assembly patterns play a central role in nanotechnology, and in particular in the optoelectronic properties of semiconductors. Changing the dots' size and shape gives rise to many applications that permeate our daily lives, such as the new Samsung QLED TV monitor that uses quantum dots to turn "light into perfect color"!

Quantum dots are obtained via the deposition of a crystalline overlayer (epitaxial film) on a crystalline substrate. When the thickness of the film reaches a critical value, the profile of the film becomes corrugated and islands (quantum dots) form. As the creation of quantum dots evolves with time, materials defects appear. Their modelling is of great interest in materials science since material properties, including rigidity and conductivity, can be strongly influenced by the presence of defects such as dislocations.

In this talk we will use methods from the calculus of variations and partial differential equations to model and mathematically analyze the onset of quantum dots, the regularity and evolution of their shapes, and the nucleation and motion of dislocations.

Friesecke, Gero

Quantum correlations and optimal transport

I will begin by explaining an interesting link - recently discovered in [1] - between quantum correlations and optimal transport, without assuming familiarity with either. More precisely, in a suitable dilute scaling limit, position correlations between N quantum particles (say, electrons in a molecule) are described by a Kantorovich optimal transport problem. Unlike in usual OT problems (as discussed in the books by Villani), one needs to prescribe N instead of 2 marginals (the probability distributions of the individual electrons), and the cost is not a positive but a negative power of the distance (-1, corresponding to Coulomb repulsion, instead of 2).

Next, I will explain that (and why) for OT problems with more than 2 marginals, the familiar two-marginal result that "Monge = Kantorovich" fails, i.e. the infimum of the Monge problem can be strictly bigger than the minimum of the Kantorovich problem.

I will close by presenting a new ansatz [2] for the discretized multi-marginal OT problem which - unlike the Monge ansatz - always yields the minimum Kantorovich cost, but whose computational complexity scales linearly instead of exponentially with the number of particles/marginals.

Joint work with Claudia Klueppelberg (TUM), Codina Cotar (UCL), Brendan Pass (Alberta), Huajie Chen (Beijing Normal), Daniela Voegler (TUM).

Main references:
[1] C.Cotar, G.F., C.Klueppelberg, Density functional theory and optimal transportation with Coulomb cost, Comm. Pure Appl. Math. 66, 548-599, 2013
[2] G.F., D.Voegler, Breaking the curse of dimension in multi-marginal Kantorovich optimal transport on finite state spaces, arXiv 1801.00341, 2017

Hastings, Stuart

On the use of classical ode methods in modern applied analysis

We give several examples of physically important ode boundary value problems for which existence of solutions has been proved in the literature by at least two distinctly different methods, one of which can be called a "PDE method", or perhaps a "dynamical systems" method, and one of which uses classical ode techniques, such as the stable manifold theorem. For these examples the ode methods have some significant advantages. One moral is that PDE students, who may well encounter nontrivial ode problems in their research, should study ODEs beyond the basic techniques of a first course.

If time permits, three important unsolved problems in the area will be presented.

Niethammer, Barbara

Instabilities and oscillations in coagulation equations

Smoluchowski's classical coagulation equation can describe mass aggregation phenomena in a large variety of applications, such as aerosol physics, polymerization, population dynamics, or astrophysics. The model consists of a nonlinear integral equation for the number density of clusters and involves a rate kernel that accounts for the microscopic details of the specific aggregation process. Of particular relevance is to understand whether the long-time behaviour of solutions is universal. In this talk I will in particular discuss the case of diagonally dominant kernels of homogeneity one. Here formal arguments lead to the conjecture that for large times the coagulation equation can be seen as a regularization of the Burgers equation. In contrast to diffusive regularization, however, we obtain phenomena such as instability of the constant solution or oscillatory traveling waves.

Pego, Robert

Self-similar limits from ballistic annihilation of fronts

We study self-similar limits in a coagulation-growth model motivated by the large-scale dynamics of bistable reaction-diffusion equations in one dimension. We identify criteria for convergence to self-similar form, in terms of regular variation, that apply in many circumstances, but also exhibit counterexamples. This is joint work with Jack Carr.

Penrose, Oliver

An application of centre manifold theory in economics

Thirty-five years ago, when most economists were extolling the virtues of financial deregulation and innovation, a maverick named Hyman P. Minsky (1919-1986) maintained a more negative view of the financial world : he noted that bankers, traders, and other financiers periodically played the role of arsonists, setting the entire economy ablaze. And later on, the financial crash of 2007-2008 came to be described as a "Minsky moment". Minsky did not put his ideas into mathematical form but a "monetary Minsky model" for this purpose has been advanced by Steve Keen (1953-). I will show how some of the nonlinear analysis in Jack Carr's book "Applications of centre manifold theory" helps us to understand the performance of Keen's model. (This is joint work with Prof. Paul Hare of the Heriot-Watt University School of Social Sciences)

Slemrod, Marshall

From C-G-S to Hilbert 6

In the early 1980's I had the privilege of collaborating with Jack Carr and Mort Gurtin on application of the van der Waals–Korteweg second gradient theory of materials to phase transitions on elastic bars and compressible fluids. I had no appreciation at the time that this C-G-S work would be a theme that I would return to throughout my career. But one thing that always bothered me was the lack of a reductionist derivation of such a second gradient theory, say from Boltzmann's kinetic theory of gases. It was then in the not too distant past that the work of Alexander Gorban and Ilya Karlin came to my attention and at least in some simplified situations one can see a direct map from the mesoscopic theory of Boltzmann to a continuum the theory of gases which includes Korteweg's second gradient theory.

Furthermore this map to some extent answers the question posed by Hilbert in his sixth problem, namely develop "mathematically the limiting processes ... which lead from the atomistic view to the laws of motion of continua." As a side benefit of this map one can easily see that the goal of actually obtaining the classical Euler (or even Navier–Stokes equations) for a compressible fluid via a limit from the Boltzmann equation cannot be achieved.

Stewart, Iain

A continuum model for smectic A liquid crystals

Smectic liquid crystals are anisotropic fluids consisting of rod-like molecules which are arranged in equidistant layers. In smectic A (SmA) liquid crystals these molecules have a common preferred local average direction parallel to the local layer normal. The average orientation of their long molecular axes is described by the unit vector n, called the director. The unit smectic layer normal is denoted by a and it is common to make no distinction between n and a in static configurations. Most previous dynamic continuum theories for SmA also suppose that n and aare always coincident. However, the dynamics of SmA materials is quite complex: work by Auernhammer and co-workers indicates that SmA under simple shear may exhibit a decoupling between n and a, thereby necessitating a theory that will reflect this effect and its consequences. The usual Oseen constraint ($\nabla\times$ a = 0) for smectics can be accommodated within this theory, but it is not necessarily imposed upon the unit normal a. Permeation effects are also included.

Székelyhidi, László

High-dimensionality and h-principle in PDE

In this talk I would like to present an analyst's point of view on the famous Nash–Kuiper Theorem and, in particular, highlight the very close connection to turbulence – a paradigm example of a high-dimensional phenomenon. The aim is to explain recent applications of Nash's ideas to the incompressible Euler equations and the recent resolution of Onsager's conjecture on the energy dissipation in 3D turbulence.

Troy, William C

Periodic wave solutions of a two space dimensional neuronal model

We investigate the behaviour of solutions of a two dimensional (plus time) neuronal model which is an extension of the 1972-1973 Wilson–Cowan non-local system. We show how different types of initial conditions can evolve into rotating waves, and also outgoing periodic waves. We show how these results led to the discovery of similar wave phenomena in rat brain.