Thermodynamic formalism in dynamical systems

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Abstracts

Baladi, Viviane

On the measure of maximal entropy of Sinai billiards (joint with Mark Demers)

Sinai billiards maps and flows are uniformly hyperbolic and volume preserving – however, grazing orbits give rise to singularities. Most existing works on the ergodic properties of billiards are about the invariant volume, for which exponential mixing is known (both in discrete and continuous time). Another equilibrium state is the measure of maximal entropy. Since the discrete-time billiard is discontinuous, the mere existence of this measure is not granted a priori. I will present recent and ongoing work with Mark Demers.

Berger, Pierre

Emergence of non-ergodic, conservative dynamics

See pages 5 and 6

Bochi, Jairo

Optimization of Lyapunov exponents

I'll explain some basic results about optimization of Lyapunov exponents of linear cocycles, comparing them with corresponding results on optimization of ergodic averages.

Bruce, Catherine

Projections of Gibbs measures on self-conformal sets

Hochman and Shmerkin used Furstenburg's theory of CP-processes to prove strong Marstrand results for self-similar sets and measures with dense rotations which satisfy the strong separation condition. That is, to prove that the Hausdorff dimension of the projections of such sets and measures is the maximum possible value for every projection. Here we extend such a result to Gibbs measures on self-conformal sets without requiring any separation condition. The extension relies on some careful estimates of the entropy growth of non-linearly zoomed-in measures and the distortion of conformal iterated function systems under orthogonal projections. The result applies to Gibbs measures on hyperbolic Julia sets.

Burns, Keith

Unique equilibrium states for geodesic flows in non-positive curvature

This talk describes joint work with Vaughn Climenhaga, Todd Fisher and Dan Thompson. We consider the geodesic flow of a compact rank one manifold with non-positive curvature. Rank one means that dynamics is hyperbolic along most geodesics. The case of negative curvature, when this hyperbolicity is uniform and the geodesic flow is Anosov, was exhaustively studied by Bowen and Ruelle. We are able obtain extensions of their results, even though the hyperbolicity is non-uniform in our situation. Our proofs are based on the method of Climenhaga and Thomspson, which builds on Bowen's approach but can used in more general settings. The talk is very closely related to the immediately following talk of Nyima Kao. He will describe the extension of our results to the still broader class of manifolds with no focal points. I will aim to give an overview of the geometrical background to the two talks and introduction to how the Climenhaga–Thompson program can be implemented in the setting of geodesic flows.

Climenhaga, Vaughn

Leaf measures, dynamical dimensions, and equilibrium states

Given a transitive Anosov diffeomorphism and a Holder potential function, the conditional measures on unstable leaves for the unique equilibrium state have the property that the potential governs how they transform under iteration and under stable holonomy. I will discuss the problem of going in the other direction, constructing equilibrium states by first producing leaf measures with the right scaling properties. For the geometric potential function, leaf volume has the right scaling properties, and the equilibrium state is the SRB measure. For the zero potential, Margulis produced the appropriate leaf measures and used them to build the measure of maximal entropy. I will describe recent work with Yakov Pesin and Agnieszka Zelerowicz, in which we use the Pesin–Pitskel' dimensional characterization of topological pressure to define the appropriate leaf measures for any Holder potential. These leaf measures can be used to write down a formula for the unique equilibrium state, giving a generalization of Margulis's result. Applying this procedure to the geometric potential produces an explicit formula for the SRB measure.

Dobbs, Neil

Free energy jumps up

In joint work with Mike Todd, we study the dynamics of smooth interval maps. Given a convergent sequence of ergodic invariant probability measures, we examine the limit behaviour of metric entropy, Lyapunov exponents and the free energy (for geometric potentials). A new and fairly canonical way of inducing gives allows us to obtain tighter estimates on limit measures. As a corollary, under suitable hypotheses, equilibrium states vary continuously as we vary the map and the potential.

Dougall, Rhiannon

Counting for amenable/non-amenable group extensions of Anosov flows

Recent work for geodesic flows (for compact negatively curved manifolds) states that the exponential growth rate of periodic orbits in a G-cover is equal to the topological entropy of the base if and only if G is amenable. As the tools are symbolic dynamics, one might expect this to extend to Anosov flows. However, key to the geodesic flow case, is the presence of the time reversal involution $(x, v) \to (x, -v)$. We discuss our results in the context of Anosov flows. This is joint work with R. Sharp.

Fraser, Jonathan

Dynamics on fractals (PUBLIC LECTURE)

Roughly speaking, a 'fractal' is an object which exhibits complicated behaviour at arbitrarily small scales. Fractals arise naturally across mathematics and wider science and, due to their complicated structure, they can be difficult to study. They often appear as 'dynamical invariants' and if the ambient dynamical system is well-understood, then one can appeal to powerful tools from ergodic theory. I will discuss some problems related to this heuristic.

Gouezel, Sebastien

Ruelle resonances for linear pseudo-Anosov maps

The Ruelle resonances of a dynamical system are spectral characteristics of the system, describing the precise asymptotics of correlations. While one can usually show their existence by abstract spectral arguments, they are most of the time not computable. I will explain that, in the case of linear pseudo-Anosov maps, one can describe them explicitly in terms of the action of the pseudo-Anosov on cohomology. Joint with Frédéric Faure and Erwan Lanneau.

Iommi, Godofredo

Upper semi-continuity of the entropy map for Markov shifts

In this talk I will show that for finite entropy countable Markov shifts the entropy map is upper semi-continuous when restricted to the set of ergodic measures. This is joint work with Mike Todd and Aníbal Velozo.

Jézéquel, Malo
Distribution of Ruelle resonances for non-analytic hyperbolic diffeomorphisms
Given a smooth uniformly hyperbolic diffeomorphism and a weight, one may define their Ruelle resonances that are the relevant eigenvalues of a transfer operator associated to the weighted dynamics. These resonances describe accurately the rate of mixing of some Gibbs measure. From the work of Ruelle, Rugh and Fried, it is known that real analytic diffeomorphisms have "few" resonances. We shall explain how to construct smooth diffeomorphisms and weights that give rise to many Ruelle resonances and then discuss the case of Gevrey dynamics whose behavior is in a way similar to the real-analytic case.

Jurga, Natalia
A dimension gap for Bernoulli measures for the Gauss map
See page7

Kaenmaki, Antti
Subadditive ergodic theory and applications
This talk serves as an introduction to subadditive ergodic theory and thermodynamical formalism. The main motivation for developing such theory comes from the submultiplicativity of the operator norm. Conversely, the main applications of the theory are random matrix products, thermodynamic formalism for matrix-valued potentials, and dimension theory for self-affine sets. In the talk, I give a brief historical account for the topic, review some of the recent advances, and exhibit possible directions for future work.

Gelfert, Katrin
Weak- and in entropy approximation of non-hyperbolic measures in partially hyperbolic diffeormorphisms*
We consider certain partially hyperbolic non-hyperbolic diffeomorphisms with one-dimensional central bundle and study how non-hyperbolic ergodic measure (i.e. with zero central exponent) can be approximated in the weak* topology and in entropy by measures supported in basic sets with positive (negative) central Lyapunov exponent. Our method also allows to show how entropy changes "across" measures with central Lyapunov exponent close to zero. Besides the construction of ergodic theoretic skeletons, our arguments are mainly based on the existence of minimal strong foliations and the existence of blender-horseshoes. We also prove that any non-hyperbolic ergodic measure is in the intersection of the convex hulls of the measures with positive central exponent and with negative central exponent. This talk is based on joint work with L.J.Díaz, M.Rams, and B.Santiago.

Kao, Nyima
Unique equilibrium states for geodesic flows on surfaces without focal points
It is well-known that for compact uniformly hyperbolic systems Holder potentials have unique equilibrium states. However, it is much less known for non-uniformly hyperbolic systems. In his seminal work, Knieper proved the uniqueness of the measure of maximal entropy for the geodesic flow on compact rank 1 non-positively curved manifolds. A recent breakthrough made by Burns, Climenhaga, Fisher, and Thompson which extended Knieper's result and showed the uniqueness of the equilibrium states for a large class of non-zero potentials. This class includes scalar multiples of the geometric potential and Holder potentials without carrying full pressure on the singular set. In this talk, I will discuss a further generalization of these uniqueness results, following the scheme of Burns–Climenhaga–Fisher–Thompson, to equilibrium states for the same class of potentials over geodesic flows on compact rank 1 surfaces without focal points. This work is an MRC project joint with Dong Chen, Kiho Park, Matthew Smith, and Regis Varao.

Kempton, Tom
On the Hausdorff dimension of Bernoulli convolutions
Bernoulli convolutions are a simple family of self-similar measures with overlaps. The problem of determining which parameters give rise to Bernoulli convolutions of dimension one has been studied since the 1930s, and is still far from being completely solved. For algebraic parameters, we show how to give an expression for the dimension of the Bernoulli convolution in terms of products of matrices. This allows us to conclude that the Bernoulli convolution has dimension one in many examples where the dimension was previously unknown. This is joint work with Shigeki Akiyama, De-Jun Feng and Tomas Persson.

Lim, Seonhee
Hausdorff dimension of bad sets in inhomogeneous Diophantine approximation
We show that for any $e > 0$, and almost every vector x, the set of e-bad vectors

$$w: liminf\ q^{1/n} \|qx - w\| > e$$

has Hausdorff dimension strictly less than n. We will further give a sufficient condition on x which ensures the above result, and will give the necessary and sufficient condition for dimension 1. We will explain the dynamical proof which uses the fact that maximality of relative entropy of a diagonal flow implies invariance under translations. The main part of the talk is a joint work with Uri Shapira and Nicolas de Saxcé.

Lima, Yuri
Symbolic dynamics for non-uniformly hyperbolic systems with singularities
Symbolic dynamics is a tool that simplifies the study of dynamical systems in various aspects. It is known for almost fifty years that uniformly hyperbolic systems have “good” codings. For non-uniformly hyperbolic systems, Sarig constructed in 2013 “good” codings for surface diffeomorphisms. In this talk we will discuss some recent developments on Sarig's theory, when the map has discontinuities and/or critical points, such as multimodal maps of the interval and Bunimovich billiards.

Pollicott, Mark
Thermodynamic formalism and the Zaremba conjecture
The Zaremba conjecture deals with the denominators of finite continued fractions with bounded digits. Although the conjecture remains open substantial progress was made by Bourgain–Kontorovich and Huang. An interesting ingredient is thermodynamic and we will describe a rigorous approach to it.

Schapira, Barbara
Strongly positively recurrent manifolds and applications
After a brief presentation of a geometric construction of equilibrium measures of the geodesic flow in negative curvature, and of a finiteness criterion of these measures, I will present a wide class of non-compact manifolds, called SPR (Strongly Positively Recurrent), on which we establish the differentiability of topological entropy along small C^1-perturbations of the metric. It generalizes an older result of Katok–Knieper–Weiss in the nineties. The results presented are mainly in collaboration with Samuel Tapie, but also Frederic Paulin, Mark Pollicott, and Vincent Pit.

Terhesiu, Dalia
Limit laws along subsequences for null recurrent renewal shifts
In the first part of the talk, I will recall some classical limit laws (including mixing) along the full sequence. In the second part, I will talk about limit laws along subsequences focusing on an analogue of the Darling–Kac law (including properties of the limit distribution) and a Karamata type theorem. The latter is based on joint work with P. Kevei.

Thomsen, Klaus
KMS states, conformal measures and ends in digraphs
I will present recent work on KMS states for certain one-parameter groups on C*-algebras and the conformal measures for the shift on the pathspace of a digraph.

Emergence of non-ergodic, conservative dynamics

Pierre Berger, with two works in progress:
one with Jairo Bochi and one with Dimitry Turaev

The Birkhoff ergodic theorem states that given an ergodic probability measure μ, for μ-almost every point x, the Birkhoff average:

$$S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$$

converges to μ. For many differentiables maps f, there are finitely many ergodic probability measures $(\mu_i)_i$ so that for Lesbesgue a.e. $x \in M$, $S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$ converges to one of the μ_i. Interestingly, this class of systems contains some of those with positive entropy (system for which it is impossible to predict the precise position of many orbits).

However, it has been shown in [Be16], that this paradigm is not sufficient to describe many typical dynamical systems. More precisely, we showed that among some open subsets of differentiable dynamical systems of a compact manifold M, typically in the sense of Kolmogorov, a system displays infinitely many attractors with very different statistical behaviors. To describe the complexity of such dynamics, the following notion has been introduced in [Be17]:

Definition 1. *Given a standard distance on the probability measures of a space (such as the Wasserstein metric), the Emergence $\mathcal{E}(\epsilon)$ at scale $\epsilon > 0$ of a system is the minimal number N of probability measures $(\mu_i)_{1\le i\le N}$ necessarily so that the Birkhoff average $S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$ satisfies:*

$$\limsup_{k\to\infty}\int_M d_{W_1}(S_k(x),\{\mu_i : 1\le i\le N\})dLeb < \epsilon\ ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures of M.

In [Be17], it has been conjectured:

Conjecture 2. *In many categories of dynamical systems, a typical dynamics displays a super polynomial emergence:*

$$\limsup_{\epsilon\to 0} \frac{\log \mathcal{E}(\epsilon)}{-\log \epsilon} = \infty \ .$$

We will present recent developments on this program, and its analog with the theory of systems of positive entropy (including the positive entropy conjecture [BT17]).

In a work in progress with Jairo Bochi, we showed an analog of the variational principle of the entropy for the concept of emergence. Furthermore, we showed that in the open set of conservative, surface mapping diffeomorphisms displaying an elliptic point, a C^∞-generic diffeomorphism displays a maximal emergence (which is super-polynomial):

$$\limsup_{\epsilon\to 0} \epsilon^2 \log \mathcal{E}(\epsilon) > 0 \ .$$

In a work in progress with D. Turaev, we showed that in the open set of Hamiltonian diffeomormhisms with a totally elliptic point, a typical diffeomorphism in the sense of Kolmogorov (i.e. Lebesgue a.e. map in a generic family) displays a maximal emergence:

$$\limsup_{\epsilon\to 0} \epsilon^{2n} \log \mathcal{E}(\epsilon) > 0 \ .$$

This proves this conjecture in the category of Hamiltonian (and surface, conservatif) context displaying an elliptic fixed periodic point.

References

[Be16] Berger, P. – P. Berger, Generic family with robustly infinitely many sinks, *Inventiones Matematicae*, (2016) 205 : 121. 41p.

[Be17] Berger, P. – Emergence and non-typicality of the finiteness of the attractors in many topologies *Proc. Steklov Inst. Math.* 297 (2017), no. 1, 1–27.

[BT17] Berger, P. and Turaev, D. – On Herman Positive entropy conjecture *Arxiv* (2017).

A DIMENSION GAP FOR BERNOULLI MEASURES FOR THE GAUSS MAP

NATALIA JURGA

Abstract. It is well known that the Gauss map $T : [0,1] \setminus \mathbb{Q} \to [0,1] \setminus \mathbb{Q}$ given by

$$T(x) = \frac{1}{x} \mod 1$$

has an absolutely continuous invariant probability measure μ_T given by

$$\mu_T(A) = \frac{1}{\log 2} \int_A \frac{1}{1+x} dx.$$

Let $\mu_{\mathbf{p}}$ denote the Bernoulli measure associated to the countable probability vector $\mathbf{p}$, projected to $[0,1]$ in the usual way. Kifer, Peres and Weiss showed that the Bernoulli measures for the Gauss map satisfy a *dimension gap* meaning that there exists $c > 0$ such that

$$\sup_{\mathbf{p}} \dim_{\mathrm{H}} \mu_{\mathbf{p}} < 1 - c. \tag{0.1}$$

Moreover, they showed that $c \geq 10^{-7}$. Their proof was based on considering sets of large deviations for the asymptotic frequency of certain digits from the one prescribed by μ_T. In this talk we will discuss an alternative proof of (0.1) which instead reduces to obtaining good lower bounds on the asymptotic variance of a class of potentials..