REPORT ON THE CONFERENCE MOTIVIC INTEGRATION AND ITS INTERACTIONS WITH MODEL THEORY AND NON-ARCHIMEDEAN GEOMETRY

ICMS, 14, India Street, Edinburgh

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Organized by: Raf Cluckers, Angus Macintyre, Johannes Nicaise and Julien Sebag.

1. SCIENTIFIC REPORT

The objective of this workshop has been to realize progress on Motivic Integration by gathering some of the leading specialists active in and around this domain and neighbouring domains such as Rigid Geometry and Model Theory. Since its creation by M. Kontsevich in 1995, Motivic Integration has been a rapidly developing subject connected to Algebraic Geometry, Singularity Theory, Model Theory and Number Theory. Because of the many challenges related to the development of the theory and its applications and the rapid evolution of the subject, this meeting should yield substantial new developments and open up many new challenges.

The talks were divided into two parts:

- Short courses. Their aim was to give an introduction to the following subjects: Non-archimedean Geometry (Bosch (2×1h), Berkovich (2×1h), Huber (2×1h)), Model Theory (Macintyre (1h), Chatzidakis (1h), Cluckers (1h)), Motivic integration (Denef (1h), Nicaise (2×1h), Scanlon (1h), Loeser (1h));
- Specialized talks (50 minutes), with a special emphasis on applications in singularity theory (Merle, Temkin, Veys) and the Langlands program (Fargues, Hales, Kaletha).

The short courses were of particular importance because they offered a survey of some recent developments in domains in full expansion. In particular, it was to our knowledge the first time that all major approaches to non-archimedean geometry (rigid varieties, formal schemes, analytic spaces, adic spaces, Zariski-Riemann spaces) were presented and explained at the same conference; the short course on motivic integration covered the various formalisms developed since Kontsevich' original work (geometric motivic integration on algebraic varieties, formal schemes and non-archimedean spaces; model-theoretic framework of Cluckers-Loeser and Hrushovski-Kazhdan). Since these developments are not cumulative and each of the theories presents its own merits in particular applications, it was quite fruitful to introduce them all and to allow for a comparison of their specific characteristics.

Moreover, each of the short courses was taught by a leading specialist; the presence of all these experts stimulated discussions and exchange of ideas, and encouraged young researchers to ask for suggestions and advice. It has been our explicit aim to cross some of the artificial boundaries between different branches of mathematics and to enhance interdisciplinary collaboration. The combination of short courses and specialized talks made it possible to appeal at once to students and experts, and to cover a wide area of subtly connected topics in an accessible way.

The answers to the questionnaires have confirmed the value of this approach, and underlined the high quality of the talks. Several participants indicated that the conference resulted in new collaborations and ideas for further research. Some excerpts:

"Very high scientific level, but nevertheless accessible."

"The presentations of the different approaches to rigid geometry were excellent and very useful. I was also personally very happy to meet for the first time some researchers whose work I had had the occasion to apply in my own work, and to exchange ideas with them."

“In my view, the workshop was a very interesting one, since it concentrated on several different fields, whose relations among each other had not been observed to a greater extent so far.”

(on the academic value of the meeting:) “It was very considerable, with an unusual mix of mathematicians from different but subtly interconnected areas. I felt that the talks were of very high quality, and the interactions valuable.”

“First opportunity to meet p-adic analytic geometers of all kind together.”

“I think that the workshop contributed a lot to the interaction between few different areas which have some touch in the objects they are studying but use very different techniques and languages in their research.”

“The workshop provided an excellent overview over the various fields and their interactions. The specialized talks in my own field drew my attention to interesting details, and they gave me inspiration for my own research. So did the discussions during the breaks.”

2. ORGANIZATIONAL REPORT

All participants (and organizers) were very pleased with the excellent organization by the ICMS staff; we thank them warmly for their work. Everybody enjoyed the conference dinners and the availability of tea and coffee during the breaks; these elements contributed to the informal character of the meeting and created a fertile atmosphere for scientific discussions.

LIST OF PARTICIPANTS :

1. **Aschenbrenner, Matthias,** University of California, Los Angeles
2. **Balwe, Chetan,** University of Pittsburgh
3. **Berkovich, Vladimir,** Weizmann Institute of Science
4. **Blickle, Manuel,** University of Essen
5. **Bosch, Siegfried,** Westfälische Wilhelms - Universität Münster
6. **Chambert-Loir,** Antoine, Université Rennes 1
7. **Chatzidakis, Zoé,** CNRS - Université Paris 7
8. **Cluckers, Raf,** École Normale Supérieure (ENS Paris)
9. **Comte, Georges,** Université de Nice - Sophia Antipolis
10. **Delon, Françoise,** CNRS - Université Paris 7
11. **Denef, Jan,** University of Leuven
12. **Ducros, Antoine,** Université de Nice – Sophia Antipolis
13. **Fargues, Laurent,** Université de Nice – Sophia Antipolis
14. **Fichou, Goulwen,** Université Rennes 1, France
15. **Hales, Thomas,** University of Pittsburgh
16. **Halupczok, Immanuel,** École Normale Supérieure (ENS Paris)
17. **Huber, Roland,** Bergische Universität Wuppertal
18. **Kaletha, Tasho Statev,** University of Chicago
19. **Kappen, Christian,** Westfälische Wilhelms - Universität Münster

20. **Kato, Fumiharu,** Kyoto University

21. **Kowalski, Emmanuel,** ETH Zürich

22. **Loeser, François,** École Normale Supérieure (ENS Paris)

23.**Macintyre, Angus,** University of London

24.**Macpherson, Dugald,** University of Leeds

25.**Merle, Michel,** Université de Nice - Sophia Antipolis

26.**Morrow, Matthew,** University of Nottingham

27. **Nicaise, Johannes,** Université Lille 1

28. **Pillay, Anand,** University of Leeds

29. **Raibaut, Michel,** Université de Nice - Sophia Antipolis

30. **Rökaeus, Karl,** Stockholm University

31. **Scanlon, Thomas,** University of California, Berkeley

32. **Sebag, Julien,** Université Bordeaux 1

33. **Soibelman, Yan,** Kansas State University

34. **Temkin, Michael,** University of Pennsylvania

35. **van den Dries, Lou,** University of Illinois

36. **Veys, Willem,** Katholieke Universiteit Leuven

37. **Wahle, Christian,** Westfälische Wilhelms - Universität Münster

LIST OF TALKS :

1. **Berkovich, Vladimir,** *Non-Archimedean analytic space*: In this series of two talks I intend to explain the notion of non-Archimedean analytic spaces, their relation to rigid analytic spaces, basic ideas of etalecohomology for them, and some of their applications.

2. **Bosch, Siegfried,** *Introduction to formal and rigid geometry:* We will give an introduction to the theory of classical rigid spaces, as invented by Tate. In addition, we will cover the approach by Raynaud via formal schemes.

3. **Chambert-Loir, Antoine,** *Igusa zeta functions in Diophantine geometry:* Recent work on Manin's conjecture concerning the number of points of bounded height in algebraic varieties and its asymptotic behaviour have shown the importance of a geometric analogue, namely the asymptotic behaviour of the volume of adelic subspaces defined by some kind of height inequalities. This behaviour has been mostly studied in the framework of algebraic groups. We explain how the analytic behaviour of analogues of Igusa zeta functions, together with Tauberian theorems, allow to recover these properties, proving them in a very general geometric situation. This is joint work with Yuri Tschinkel.

4. **Chatzidakis, Zoé,** *The basic model theory of valued fields:* I present for nonlogicians the basic concepts of the model theory of valued fields, including several of the formalisms one may use.

5. **Cluckers, Raf,** *Integration through cell decomposition and b-minimality:* I will present an introduction to cell decomposition and what it can do for p-adic and motivic integrals, and what it can not yet do for such integrals. Next I will present an axiomatic framework, which is joint work with F. Loeser, and which is built up around cell decomposition. I will comment on how it is directed towards integration and sketch some (new) open problems.

6. **Delon, Françoise,** *C-minimal structures:* A C-relation is a ternary relation first-order interpreting a tree which is a meet-semi-lattice, the domain of the C-relation being then a covering set of branches with no isolated branch. A set M, equipped with a C-relation and possibly an additional structure, is called Cminimal if any definable subset of M is definable without quantifiers in the pure language of C, and if the same holds in any elementarily equivalent structure. A C-minimal structure is algebraically bounded in the sense that finite uniformly definable sets have a bounded size. On the other hand the algebraic closure needs not satisfy the exchange principle. C-minimal structures with exchange are “geometric” in the sense of Zilber. In this context, we may ask the question of the trichotomy: Is it possible to define a group in modular non trivial structures? To define a field in non locally modular structures? To classify some of these structures?

7. **Denef, Jan,** *Course on motivic integration I:* We will look at the prehistory of motivic integration, from finite counting questions raised by Borevich - Shafarevich, Serre - Oesterlé, to p-adic integrals and to (p-adic and topological) zeta-functions. We will indicate how questions on Betti numbers of Calabi - Yau varieties led to the first approach to motivic integration by Kontsevich and how it was implemented by Denef - Loeser.

8. **Ducros, Antoine,** *The image of a flat map in non-Archimedean geometry: description via quantifier elimination for algebraically closed valued fields:* By transposing his (and Gruson's) scheme-theoretic flattening techniques to formal schemes, Raynaud proved that the image of a flat map between affinoid spaces is a union of affinoid domains. Answering a question by Berkovich, we will give a new proof of this result which doesn't use any formal model. It is based upon direct application of Raynaud and Gruson's dévissages methods in the context of (Berkovich) analytic geometry, Temkin's local study of analytic spaces through Riemann-Zariski spaces, and quantifyers elimination for algebraically closed valued fields.

9. **Fargues, Laurent,** *On some results about the étale cohomology of rigid analytic spaces linked to the local Langlands correspondence:* I will speak about three results on the étale cohomology of rigid analytic spaces linked to the geometric realization of the local Langlands correspondence in the cohomology of the Lubin-Tate and Drinfeld towers. The first one is the invariance under formal completion of the monodromy filtration of the l-adic vanishing cycles sheaves. The second one concerns an equivariant Poincaré duality theorem for the compactly supported l-adic cohomology of rigid analytic spaces, where duality has here to be understood in the sens of the Zelevinsky involution in the category of smooth representations of reductive padic groups. The last one deals with the comparison of the cohomology of the Lubin-Tate and Drinfeld tower with application a geometric form of the Jacquet-Langlands correspondence: a topos equivalence between some étale equivariant sheaves on the Drinfeld spaces and some étale equivariants sheaves on a rigid analyitic Severi-Brauer variety.

10. **Hales, Thomas,** *Transfer principle for the fundamental lemma:* This talk will explain how the identities of the fundamental lemma (arising in the theory of trace formula for automorphic representations) fall within the scope of the transfer principle, a general result that allows to transfer theorems about identities of p-adic integrals from one collection of fields to others. In particular, once the fundamental lemma has been established for one collection of fields (for example, fields of positive characteristic), it is also valid for others (fields of characteristic zero).

11. **Halupczok, Immanuel,** *Trees of definable sets in Z_p:* To a variety V defined over the p-adic integers Z_p, one can naturally associate a tree: the nodes at depth k are the points with values in Z_p/(p^k Z_p), and the tree structure is given by the canonical maps Z_p/(p^(k+1) Z_p) -> Z_p/(p^k Z_p). A variant of this also permits to replace V by arbitrary definable subsets of Z_p^n. There are old results on the number of nodes of these trees at each depth. A natural question (posed to me by Loeser) is whether these results can be strenghened to result about the structure of the trees. The goal of this talk is to present a conjecture which gives such a description. The conjecture is true for curves and for arbitrary definable subsets of Z_p^2.

12. **Huber, Roland,** *Analytic adic spaces:* We will explain the definition of analytic adic spaces. The topological spaces underlying affinoid analytic adic spaces are spaces of continuous valuations of topological rings. We will explain the relation of analytic adic spaces to classical rigid geometry and some aspects of the etale cohomology of analytic adic spaces.

13. **Kaletha, Tasho Statev,** *The stabilization of the trace formula and the fundamental lemma:* The trace formula is an important tool in the study of automorphic representations. Many applications of the trace formula, like the comparison of automorphic representations for inner forms, require that it be stabilized, and the process of stabilization inevitably leads to the need for establishing a certain identity of p-adic integrals, called the fundamental lemma, which, despite its name, had remained an open problem for multiple decades. This talk will be an overview of the stabilization process and the fundamental lemma.

14. **Kato, Fumiharu,** *Topological rings in rigid geometry:* I would like to explain our attempt to give an "umbrella notion" for topological rings in rigid geometry, which affords classical ones (type (V) and type (N)) and assists developing "absolute" notion of rigid spaces. Joint-work with Kazuhiro Fujiwara.

15. **Kowalski, Emmanuel,** *Algebraic exponential sums and their uses in analytic number theory:* The talk will be a fairly informal survey of some of the results of analytic number theory where exponential sums over finite fields or rings arise naturally, together with some of the problems and methods suggested by these. This will include, for instance, Kloosterman sums in the circle method, sums of Kloosterman sums, exponential sums over definable subsets of finite fields, and counting problems used in sieve methods.

16. **Loeser, François,** *Motivic integration via cell decomposition and applications:* In this talk we shall present our joint work with Raf Cluckers on constructing motivic integration in the definable setting using Denef-Pas cell decomposition. We shall define constructible motivic functions and consider exponentials. We shall end by giving, as an application, our general transfer principle for identities between functions defined by non archimedean integrals.

17. **Macintyre, Angus,** *Quantifier elimination in the p-adics and their algebraic closure, and issues of uniformity:* I sketch the basic idea of such eliminations, in preparation for Cluckers's lecture on cell-decomposition.

18. **Merle, Michel,** *Nearby cycles, convolution and composition with a two variable function:* We consider a two variable polynomial function and we want to study its Milnor fibre and monodromy, Hodge spectrum,..More generally, we want to compute the nearby cycles of the composed map with a mapping to the affine plane. This problem was first addressed by Némethi (1991), and by Némethi-Steenbrink (1994-95). Joint work with Gil Guibert and François Loeser.

19. **Nicaise, Johannes,** *Non-archimedean geometry and complex singularities:* We give an introduction to the theory of motivic integration on formal schemes and rigid varieties, and its applications to motivic zeta functions. We explain how non-archimedean geometry can be used to study complex hypersurface singularities.

20. **Rökaeus, Karl,** *A version of geometric motivic integration that specializes to p-adic integration:* We give a version of geometric motivic integration, valid over any complete DVR, and with the property that when we integrate with respect to an affine space over the p-adic numbers, the integral specializes to padic integration by counting F_p-points on it. The theory develops along the same lines as geometric motivic integration. The main difference is that when the geometric motivic measure takes values in a localization of the Grothendieck ring of varieties, completed with respect to the dimension filtration, we have to complete it with respect to a stronger topology. The reason for this is that we need the counting homomorphism to be continuous. We use this to explain the phenomena that certain p-adic integrals, that we are interested in, are rational functions in p, where the rational function is independent of p. We do

this by computing the corresponding motivic integral to see that it is a rational function in L, where L is the Lefschetz class.

21. **Scanlon, Thomas,** *Integration in valued fields (after Hrushovski and Kazhdan):* I will report on the work of Hrushovski and Kazhdan developing motivic integration over algebraically closed valued fields and related structures.

22. **Soibelman, Yan,** *Motivic Donaldson-Thomas invariants:* This is a joint work with Maxim Kontsevich devoted to the counting problem of stable objects in 3d Calabi-Yau categories.

23. **Temkin, Michael,** *Desingularization of quasi-excellent schemes over Q:* Grothendieck proved in EGA IV that if any integral scheme of finite type over a locally noetherian scheme X admits a desingularization, then X is quasiexcellent, and conjectured that the converse is probably true. We prove this conjecture for noetherian schemes of characteristic zero. Namely, starting with the resolution of singularities for algebraic varieties of characteristic zero, we prove the resolution of singularities for noetherian quasi-excellent schemes over Q.

24. **Veys, Willem,** *Smallest poles of motivic zeta functions:* Let f be a regular function on an nonsingular complex algebraic variety. The space of n-jets satisfying f=0 can be partitioned into locally closed subsets which are isomorphic to a cartesian product of some variety with an affine space of appropriate dimension. This implies a divisibility property for this space of n-jets in the Grothendieck ring of varieties, and a lower bound for the smallest poles of the motivic zeta function of f.