Optimal Transportation, and Application to Geophysics and Geometry

Mike Cullen, Misha Feldman, Wilfrid Gangbo, Robert McCann and Ian Roulstone

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1 Deviations from original proposal

There were no significant deviations from the original proposal. The original proposed timetable was slightly modified to fit the availability of key speakers.

2 General description

The purpose of the meeting was to bring together experts in two areas of mathematics, namely mass transportation and geometry, with experts in geophysical fluid dynamics. Geophysical dynamics seeks to understand the evolution of the atmosphere and oceans, which is fundamental to weather and climate prediction. It does this by studying systems of equations which accurately model the large-scale behaviour of the solutions, rather than the small-scale turbulent motions. Mass transportation theory was originally developed as a way of solving optimisation problems in the areas of operations research, probability and statistics. It has more recently found a much wider range of applications, such as network design, economics, medical imaging and reflector design. A wide variety of other application areas were discussed at the workshop, such as biological aggregation, plasma physics, aerodynamics and relativistic physics. In particular, it has been shown that it can be applied to fluid dynamical problems, for instance those governing the large-scale behaviour of the atmosphere and oceans. Mass transportation theory can also be given a geometrical interpretation, which has led to important extensions in its applicability. This links to fluid dynamics through the identification of incompressible flows as geodesics on an appropriate space. Recent work suggests that the classical definition of such flows has to be relaxed in order to ensure that they can be proved to exist. Recent results were described which show that mass transportation methods may be more successful in curved spaces, such as the surface of a sphere, than in flat spaces. The meeting contained a significant training element with Rick Salmon giving a mini-course on geophysical fluid dynamics and Felix Otto giving a mini-course on mass transportation methods. Several talks and posters were presented by graduate students and beginning researchers.

3 Detailed report

This workshop was held as part of an NSF-funded Focussed Research Group programme (grant no. DMS-0354729), together with funding from the EPSRC and LMS. This joint funding allowed a workshop with 65 participants, 22 of whom were graduate students or postdoctoral research assistants or fellows.

The first main theme was to increase interaction between scientists developing the theory of optimal transportation, which is a very generic mathematical method, and the

applications to geophysics. In order to do this a selection of experts in optimal transportation was invited, together with several scientists at the more theoretical end of geophysics research, who would be able to understand the new mathematical techniques being proposed.

The second aim was to increase the interaction between optimal transportation research and geometry. There has been significant recent progress in this area, which it was hoped the workshop would publicise and build on.

Because of the interdisciplinary nature of the workshop, a significant training element was included. Two mini-courses were given, one on geophysical fluid dynamics and one on mass transportation. The period from 12.00 to 16.00 on Tuesday was used for talks by graduate students and beginning researchers. Those who did not wish to speak were invited to bring a poster, of which 5 were submitted and presented.

A measure of the success of the workshop was that all 65 participants who had registered in advance turned up; and that 50 of them were still present for the final talk.

4 Description of programme

4.1 Training and overview presentations

Rob Douglas (Aberystwyth) started the meeting with an expository talk on mass transportation. He introduced the basic concepts, and demonstrated how they could be applied to geophysical fluid dynamics and incompressible flows. Rick Salmon (Scripps) gave a mini-course of three lectures on geophysical fluid dynamics. He emphasised the importance of conservation laws, and their relation through Noether's theorem to the symmetries of the problem. The presence of stratification breaks some of the symmetries. The remaining particle relabelling symmetry is related to conservation of potential vorticity. He then showed how the combined effect of energy and potential vorticity conservation is to direct energy into larger scale structures. This behaviour is confirmed by observations of the ocean. Felix Otto (Bonn) ended the week with a mini-course on mass transportation, his main emphasis being applications to the Cahn-Hilliard equation governing evolving phase boundaries in material sciences. This model is relevant to the formation of large-scale structures because interfaces between the fluids involved have significant energy, and the energy minimising solution thus favours large scales with few interfaces. He showed how the equations could be solved exploiting the variational structure by using mass transportation, in particular the idea of a transient flow being a gradient flow towards a lower energy state.

Feedback from participants on these presentations was very favourable. Many identified one or other of the courses as the highlight of the workshop. Several participants noted that they had achieved a much greater understanding of the physics behind geophysical fluid dynamics. Others noted that they had gained valuable new understanding of mass transportation and Otto's work on gradient flows.

4.2 Optimal transportation presentations

These contained a mixture of theoretical and applied talks, illustrating the breadth of the applications of mass transportation theory. At the most applied end of the spectrum, Buttazzo (Pisa) described the application of mass transport methods to planning networks, both incrementally and using a long-term strategic approach. Plakhov (Aveiro) described methods of analysing the aerodynamic resistance of rough bodies in motion through both dense and rarefied media. The more theoretical presentations included analysis by Blower

(Lancaster) of Plancherel measures on Young diagrams, and Pratelli (Pavia) of a quantitative version of the Sobolev inequality. Savare (Pavia) described the theory of gradient flows in metric spaces, particularly Wasserstein spaces.

A theme in several presentations was the application to biological aggregation, where distributions of mixtures coarsen with time because interfaces are energetically costly. Slepcev (Carnegie-Mellon) described one such system, and related it to other models using different metrics. The evolution can be described as a gradient flow in the Wasserstein metric, and is so closely related to optimal transportation. Carlen (Georgia Tech) derived uniqueness of the phase transition profile in such models as a consequence of displacement convexity of the energy. Gonzalez (Catalunya) described an analysis of the motion of a number of systems with interfaces.

Puel (Univ Paul Sabatier) showed how mass transport methods could be applied to the relativistic heat equation. Carrillo (Barcelona) described an analysis of the Keller-Segel model showing that the long-time asymptotics could be derived using mass transport methods. Brenier (Nice) showed how the Vlasov Monge-Ampere equation can arise from a model of polymers. This equation can then be solved by mass transportation methods. Gangbo (Georgia Tech) showed how the one-dimensional Euler-Poisson system could be related to the geodesic equation on a Wasserstein space, and thus solved, and Tudorascu (Georgia Tech) analysed solutions of this equation with shocks. Gomes (Lisbon) described applications to the Hamilton-Jacobi equation.

4.3 Geophysical fluid dynamics presentations

The presentations discussed in this section are those where fluid dynamics was the main theme. The ideas of Hamiltonian structure and of an incompressible flow as a volume-preserving rearrangement were common to several of the presentations. This gave a strong link to the other themes of the workshop. Several talks could equally have been included in this or either of the adjacent sections of the report, emphasising the true interdisciplinary nature of the subject.

Ambrosio (Pisa) described the theory of incompressible flows between prescribed initial and final states. It is necessary to consider relaxations of the original Arnol'd problem which required the geodesics to be volume-preserving diffeomeorphisms. The formulations introduced by Brenier lead to a problem where the velocity at each end of the geodesic can be a probability distribution. The relaxed problem can be solved, and is closely related to mass transportation theory. A striking new results is the regularity of the associated generalised pressure. Vanneste (Edinburgh) described the flow of two-dimensional incompressible fluids in terms of diffeomorphisms, and applied it to solutions for slowly deforming domains, assuming that at each time the solution is close to a steady solution for the current domain. Morrison (Austin) described the theory of the Vlasov-Poisson system, important in plasma physics, from a Hamiltonian viewpoint, in particular solution procedures in the presence of a continuous spectrum and the statistical mechanics of such solutions. Zeitlin (ENS Paris) described solutions of the geostrophic adjustment problem, showing that in general smooth solutions do not exist in a bounded domain. The existence of adjusted states can be proved by optimal transportation methods in the non-smooth case. Helena and Milton Lopes-Filho (Campinas) described work on the evolution of a semi-geostrophic vortex sheet; the question is whether this exhibits the same behaviour as a vortex sheet in two-dimensional incompressible flow. De Oliveira (Campinas) presented a poster on the stability of Lagrangian semi-geostrophic solutions. Gancedo (Madrid) discussed the well-posedness of contour dynamics methods for two problems with sharp interfaces. Goldman (Toronto) discussed instabilities in dynamical behaviour of analytic

solutions of the 2D semi-geostrophic equations.

There were also presentations focussing on obtaining numerical simulations consistent with theory. Oliver (Bremen) discussed problems of the representation of boundary conditions in numerical methods, and their effect on the statistical mechanics of the flow. Peeters (Twente) described Hamiltonian particle-mesh methods. Dubinkina (CWI) presented a poster describing the statistical mechanics of discretisations of the two-dimensional incompressible Euler equations with different conservation properties. A further novelty was the poster presented by Lanagan (Aberystwyth) on using mass transportation ideas to verify numerical forecasts.

4.4 Geometrical presentations

This section includes talks on mass transportation where the main emphasis was on geometrical aspects.

The main highlight was the geometrical interpretation of the recent work by Ma, Trudinger and Wang. That work established sufficient conditions on the cost function for the solutions of the mass transportation problem to be smooth. The geometrical work shows that the sufficient condition for regularity of solutions is also necessary, and can moreover be phrased in terms of non-negativity of certain null sectional curvatures in a pseudo-Riemannian geometry induced by the transportation cost. Talks by Y-H Kim (Toronto), Delanoë (CNRS) and Loeper (Unversite Claude Bernard, Lyon) were related to this theme. This research very significantly advances the theory.

McCann (Toronto) described joint work with Topping (Warwick) which demonstrated that the Ricci curvature flow on a Riemannian manifold can be characterized via contractivity of the conjugate heat equation with respect to the evolving transportation distance between diffusing densities. Khesin (Toronto) and his student Lee (Toronto) described the relation between the geometry of the diffeomorphism group, used to understand incompressible flows, and the Wasserstein space generated by solutions of the optimal transport problem. In particular, the shock time in the inviscid Burger's equation, which gives the geodesic flow on the diffeomorphism group, is proportional to the curvature of the manifold of volume preserving diffeomorphisms as it embeds into the space of all diffeomophisms. Roubtsov (Angers) described the relation of Monge-Ampere equations to Kähler structures on manifolds and gave examples arising from geophysics.

5 Feedback and outcomes

Many of the participants stated afterwards that the interdisciplinary nature of the workshop had been successful, and genuine communication established between researchers in the different fields represented. For instance, it was stated that 'this is a field - for some reason - where careful thinking about the applied issues has led to significant advances in pure math'. It was suggested by two participants that additional material demonstrating exactly how optimal tranportation could be used in geophysical fluid dynamics would have been beneficial and that some structured collaboration could have been included in the timetable. It is likely that another interdisciplinary workshop will be organised by some of the participants with a focus more on the aggregation problem.

There is likely to be a significant impact within the UK. It has also been suggested that a workshop be held, funded by the EPSRC Oxford Centre on nonlinear pde's, on the application of nonlinear pde theory to geophysical fluid dynamics. The mass transportation work would be one of the main themes discussed. Work in mass transportation at Aberystwyth will increase with new appointments there, the interaction at the workshop

has aided this. Interaction between the Met Office and the University of Surrey was also advanced at the workshop.

Many participants stated that they had encountered new ideas and received input from others which would be helpful in their current work. A frequent comment was that participants had acquired a much greater understanding of recent new results. Thus there was a large educational benefit. Many participants noted the benefit of the minicourses. Other particular examples were the links between the geometry of diffeomorphisms on a manifold and fluid dynamics, and the links between recent results on regularity of mass transport and the properties of mass transport on Riemannian manifolds.

Many new possible research directions were mentioned in feedback from participants. A major theme was on the links between geometry and optimal transportation, in particular the links with Ricci flow. It was commented that 'there seem to be deep connections between mass transportation and symplectic geometry yet to be understood'. Another comment was the need to advance the computational as well as the theoretical side of mass transportation. Another theme was the need to advance from the 'ad hoc' approach used in the design of most fluid dynamics computations to a more to a more systematic approach with firm theoretical foundations.

Timetable

Sunday 15 July

18.30-19.00	***Registration & buffet,*** St Trinnean's Room, St Leonard's, Pollock Halls

Monday 16 July

08.30-09.00	Registration
09.00-10.00	**Robert Douglas** (University of Wales Aberystwyth) *Introduction to mass transportation and applications to geophysics*
10.05-11.05	**Rick Salmon** (Scripps Institution of Oceanography) *Mini-course on geophysical fluid dynamics* (1)
11.05-11.35	Coffee Break
11.35-12.20	**Guiseppe Buttazzo** (University of Pisa) *Long-term planning versus short-term planning in location problems*
12.20-14.30	Lunch Break
14.30-14.45	**Marjolaine Puel** (Université Paul Sabatier) *A mass transport approach for the relativistic heat equation*
14.50-15.05	**Yann Brenier** (CNRS Nice/Paris VI) *Polymers and optimal transportation*
15.10-15.25	**Gordon Blower** (University of Lancaster) *Young diagrams and concentration*
15.25-16.05	Coffee Break
16.05-16.50	**Wilfred Gangbo** (Georgia Institute of Technology) *Euler-Poisson systems as action-minimizing paths in the Wasserstein space*
16.55-17.40	**Eric Carlen** (Georgia Institute of Technology) *Some new displacement convex functionals and some variations on displacement convexity.*

Tuesday 17 July

09.00-10.00	**Rick Salmon** (Scripps Institution of Oceanography) *Mini-course on geophysical fluid dynamics* (2)
10.05-10.50	**José Carrillo** (Universitat Autònoma de Barcelona) *The Keller-Segel mo*del: entropy, g*eometric inequalities and mass transport*
10.50-11.20	Coffee Break
11.20-12.05	**Aldo Pratelli** (University of Pavia) *A quantitative version of the Sobolev inequality*
12.10-12.25	**Adrian Tudorascu** (Georgia Institute of Technology) *Pressureless Euler/Euler-Poisson systems via adhesion dynamics and scalar conservation laws*
12.25-14.00	Lunch Break
14.00-14.30	**Poster Session**
14.30-14.45	**Bob Peeters** (University of Twente) *Hamiltonian-based numerical methods for forced-dissipative climate prediction*
14.50-15.05	**Francisco Gancedo** (NRC Madrid) *Two contour dynamics problems in incompressible flows: the Muskat problem and the QG sharp front*
15.10-15.25	**Maria del Mar Gonzalez** (Universidad Politecnica de Catalunya) *tba*
15.30-15.45	**Dorian Goldman** (University of Toronto) *Chaotic response of the 2D semi-geostrophic equations to gentle periodic forcing*
15.50-16.05	**Paul Lee** (University of Toronto) *Hamiltonian Reductions relating to optimal mass transport*
16.05-16.35	Coffee Break and **Poster Session**
16.35-17.20	**Alexander Plakhov** (University of Aveiro) *Shapes of optimal aerodynamic resistance and optimal mass transportation*
17.25-18.10	**Dejan Slepcev** (Carnegie Mellon University) *Interfacial aspects of biological aggregation*

Wednesday 18 July

09.00-10.00	**Rick Salmon** (Scripps Institution of Oceanography) *Mini-course on geophysical fluid dynamics* (3)
10.05-10.50	**Luigi Ambrosio** (Scuola Normale Superiore di Pisa) *Variational models for incompressible Euler equations*
10.50-11.20	Coffee Break
11.20-12.05	**Jacques Vanneste** (University of Edinburgh) *Two-dimensional Euler flows in slowly deforming domains: adiabatic invariance and geometric angle*
12.10-12.25	**Helena Lopes** (University of Campinas) *On the continuous dependence with respect to initial data of Lagrangian solutions to the semigeostrophic equations*
12.30-12.45	**Milton Lopes-Filho** (University of Campinas) *On potential vorticity sheets for semigeostrophy*
12.45-14.45	Lunch Break
14.45-15.30	**Phil Morrison** (University of Texas at Austin) *Statistical mechanics of fluid and plasma continua*
15.35-16.05	Coffee Break
16.05-16.50	**Diogo Gomez** (IST Lisbon) *Linear programming and homogenization of Hamilton-Jacobi equations*
16.55-17.40	**Marcel Oliver** (Jacobs University Bremen) *Transparent boundary conditions as dissipative subgrid closures*

Thursday 19 July

09.00-10.00	**Felix Otto** (Universität Bonn) *Mini-course on some applications of optimal transportation* (1)
10.05-10.50	**Guiseppe Savare** (University of Pavia) *Gradient flows and diffusion semigroups in metric spaces under lower curvature bounds*
10.50-11.20	Coffee Break
11.20-11.35	**Philippe Delanoë** (CNRS) *On the "staying away from cut-locus" issue in smooth optimal transport*
11.40-12.25	**Gregoire Loeper** (Université Claude Bernard Lyon 1/BNP Paribas) *Continuity of maps solutions of optimal transportation problems*
12.30-14.30	Lunch Break
14.30-15.30	**Felix Otto** (Universität Bonn) *Mini-course on some applications of optimal transportation* (2)
15.30-16.00	Coffee Break
16.00-16.45	**Young-Heon Kim** (University of Toronto) *Curvature and the continuity of optimal transportation maps*
16.50-17.35	**Volodya Roubtsov** (Université d'Angers) *Geometries and generalized geometries beyond Monge-Ampère*
19.30	Workshop Dinner, St Trinnean's Room, St Leonards

Friday 20 July

09.00-10.00	**Felix Otto** (Universität Bonn) *Mini-course on some applications of optimal transportation* (3)
10.05-10.50	**Vladimir Zeitlin** (École Normale Supérieure) *Lagrangian approach to catastrophic adjustment and symmetric instability*
10.50-11.20	Coffee Break
11.20-12.05	**Boris Khesin** (University of Toronto) *Geometry of diffeomorphism groups: shocks of the Burgers equation and non-holonomic mass transport*
12.10-12.55	**Robert McCann** (University of Toronto) *Ricci flow, entropy, and optimal transportation*

Participants' Details

Name	Institution	email
Ambrosio, Luigi	Scuola Normale Superiore di Pisa	l.ambrosio@sns.it
Barrett, John	Imperial College, London	j.barrett@imperial.ac.uk
Bevan, Jonathan	University of Surrey	j.bevan@surrey.ac.uk
Blower, Gordon	University of Lancaster	g.blower@lancaster.ac.uk
Brenier, Yann	CNRS Nice/Paris VI	brenier@math.unice.fr
Buffoni, Boris	EPF Lausanne	boris.buffoni@epfl.ch
Burton, Geoffrey	University of Bath	grb@maths.bath.ac.uk
Buttazzo, Guiseppe	University of Pisa	buttazzo@dm.unipi.it
Carlen, Eric	Georgia Institute of Technology	carlen@math.gatech.edu
Carrillo, José A.	Universitat Autònoma de Barcelona	carrillo@mat.uab.es
Carvalho, Maria	University of Lisbon	mcarvalh@math.gatech.edu
Castro, Angel	CSIC Madrid	angel_castro@imaff.cfmac.csic.es
Chen, Jun	University of Wisconsin	jchen@math.wisc.edu
Cordoba, Diego	NRC Madrid	dcg@imaff.cfmac.csic.es
Cullen, Mike	Meteorological Office	mike.cullen@metoffice.gov.uk
de Oliveira, Josiane Cristina	UNICAMP	joliveira@ime.unicamp.br
Delahaies, Sylvain	University of Surrey	s.delahaies@surrey.ac.uk
Delanoë, Philippe	CNRS	delphi@math.unice.fr
Douglas, Robert	University of Wales Aberystwyth	rsd@aber.ac.uk
Dubinkina, Svetlana	CWI Amsterdam	s.dubinkina@cwi.nl
Feldman, Mikhail	University of Wisconsin-Madison	feldman@math.wisc.edu
Figalli, Alessio	Scuola Normale Superiore di Pisa	a.figalli@sns.it
Gancedo, Francisco	NRC Madrid	fgancedo@imaff.cfmac.csic.es

Name	Institution	email
Gangbo, Wilfred	Georgia Institute of Technology	gangbo@math.gatech.edu
Goldman, Dorian	University of Toronto	dorian.goldman@utoronto.ca
Gomes, Diogo	IST Lisbon	dgomes@math.ist.utl.pt
Gonzalez, Maria del Mar	University of Texas at Austin	mar.gonzalez@upc.edu
Karakhanyan, Aram	University of Texas at Austin	aram@math.utexas.edu
Khesin, Boris	University of Toronto	khesin@math.toronto.edu
Kim, Hwa Kil	Georgia Institute of Technology	hwakil@math.gatech.edu
Kim, Young-Heon	University of Toronto	yhkim@math.toronto.edu
Lanagan, Gareth	University of Aberystwyth	gdl00@aber.ac.uk
Lee, Paul	University of Toronto	plee@math.toronto.edu
Lisini, Stefano	Universita del Piemonte Orientale	stefano.lisini@unipv.it
Loeper, Gregoire	Université Claude Bernard Lyon 1/BNP Paribas	loeper@math.univ-lyon1.fr
Lopes, Helena	University of Campinas	hlopes@ime.unicamp.br
Lopes-Filho, Milton	University of Campinas	mlopes@ime.unicamp.br
Maggi, Francesco	Università degli Studi di Firenze	maggi@math.unifi.it
Markowich, Peter	Faculty of Mathematics, University of Vienna	peter.markowich@univie.ac.at
McCann, Robert	University of Toronto	mccann@math.toronto.edu
Morrison, Phil	University of Texas at Austin	morrison@physics.utexas.edu
Norbury, John	University of Oxford	john.norbury@lincoln.oxford.ac.uk
Oliver, Marcel	Jacobs University Bremen	m.oliver@jacobs-university.de
Otto, Felix	Universität Bonn	otto@iam.uni-bonn.de
Pacini, Tommaso	Imperial College London	t.pacini@imperial.ac.uk
Panike, Nathan	University of Wisconsin	panike@math.wisc.edu
Peeters, Bob	University of Twente	b.w.i.peeters@utwente.nl
Plakhov, Alexander	University of Aveiro	plakhov@mat.ua.pt
Pratelli, Aldo	University of Pavia	aldo.pratelli@unipv.it

Name	Institution	email
Puel, Marjolaine	Université Paul Sabatier	puel@mip.ups-tlse.fr
Roubtsov, Volodya	Université d'Angers	vladimir.roubtsov@univ-angers.fr
Roulstone, Ian	University of Surrey	i.roulstone@surrey.ac.uk
Salmon, Rick	Scripps Institution of Oceanography	rsalmon@ucsd.edu
Savare, Giuseppe	University of Pavia	giuseppe.savare@unipv.it
Schonbek, Maria	University of California Santa Cruz	schonbek@maths.ox.ac.uk
Slepcev, Dejan	Carnegie Mellon University	slepcev@math.cmu.edu
Sosio, Maria	University of Toronto	msosio@math.toronto.edu
Spirn, Dan	University of Minnesota	spirn@math.umn.edu
Thuburn, John	University of Exeter	j.thuburn@exeter.ac.uk
Trokhimtchouk, Maxim	University of California, Berkeley	trokhim@math.berkeley.edu
Tudorascu, Adrian	Georgia Institute of Technology	adriant@math.gatech.edu
Vanneste, Jacques	University of Edinburgh	j.vanneste@ed.ac.uk
Wirosoetisno, Djoko	Durham University	djoko.wirosoetisno@durham.ac.uk
Yolcu, Turkay	Georgia Institute of Technology	tyolcu@math.gatech.edu
Zeitlin, Vladimir	École Normale Supérieure	zeitlin@lmd.ens.fr

ICMS WORKSHOP ON OPTIMAL TRANSPORTATION, AND APPLICATIONS TO GEOPHYSICS AND GEOMETRY

JULY 2007

1. What, for you, was the highlight of the workshop?

*The short course given by F. Otto - it was just luck that this sort of thing is of current interest to me, and they were very well presented.

*The mini-courses and some talks were particularly well given, with fundamentally new ideas.

*Its interdisciplinary character which was quite timely, revealing deep and motivating links between the different areas represented: geometry, analysis and geophysical fluid dynamics.

*The lectures by Felix OTTO, which gave the right amount of detail and were of topical interest.

*The mixture of talks: they were from different areas of mathematics but all related. I met a lot of different people, learnt about new problems, and found some mathematicians with similar interests to mine that I didn't know before. This is interesting for future research.

*The talks of Brenier, Ambrosio, McCann

*Gathering and interaction of researchers with very different interests

*The interdisciplinary nature. I was exposed to a great many ideas that were new to me.

*To present the most recent and deep applications of Optimal Transportation to many area of pure and applied mathematics as geophysics, nonlinear PDE's, functional inequalities, geometry.

*Simply the opportunity to interact with theoreticians and mathematicians who are interested in fluids.

*This is was a very good workshop, with both people doing more fundamental research in optimal transport, as well as more applied problems, namely in the area of Geophysical Fluid Dynamics.

*The lecture courses by Rick Salmon introduced me to a number of very interesting ideas that I was not familiar with, and yet are close to issues that I have been working on actively. I expect it will have a direct impact on my research. But the whole program was quite interesting, and many other presentations -- Felix Otto's course, Guiseppe's Savare's talk, etc., were very interesting, as were the discussions over meals and between talks!

*The personal highlights of the conference were being able to rub shoulders with some of the greats of my field, such as Prof. Yann Brenier, Dr. Mike Cullen and Dr. Ian Roulstone to name but a few. With regards to the talks that were given, I enjoyed

Guiseppe Buttazzo's talk on "Long Term planning versus short-term planning in location problems". Furthermore, Rob Douglas' "Introduction to mass transportation and applications to geophysics" and Yann Brenier' talk on "Polymers and optimal transportation" were particular highlights.

*The links between optimal transportation and geophysical fluid dynamics.

*There were a number of excellent presentations, but the most enjoyable for me was Alexander Plakhov's talk.

*For me the highlight of the workshop was meeting of so many old and making so many new acquaintances. I was lucky to be able to meet some of the most talented people in my field.

*Geophysical fluid dynamicists and mathematicians talking to each other

*The fact that everyone had registered turned up, and that 50 out of the 64 were still there for the final talk.

*This workshop was a unique opportunity for me to gather with as many specialists of the field of optimal transportation. The scope of the workshop was very consistent, precise enough so that people could exchange on their research, and also sufficiently broad to learn about new areas that are under development. This has really been a very productive workshop as far as I am concerned.

*I enjoyed many of the talks, especially Rick Salmon's lectures, and Robert Douglas' lecture on geophysical flows and semigeostrophy. I also especially liked the talks by Luigi Ambrosio, Gregoire Loeper, Diogo Gomes and Robert McCann. The highlight was the interaction with so many of the iconic figures in this field. Interaction was most productive with Misha Feldman, Robert McCann and Geoffrey Burton.

*To meet important people in my field of research (geophys.fluid dyn.) To see some nice applications of optimal transport in geophys. fluid dyn.

*Highlights were Rick Salmon's lecture series on geophysical fluids, and the day devoted to regularity of optimal mappings (Delanoe, Loeper, Kim).

*The talks of Gangbo and Carrillo.

*Felix Otto's lectures, which gave an excellent synthesis of theory and applications.

*Gathering the very lively (and rather numerous) community of "optimal transport theory" (mostly oriented toward non-linear PDEs) and top experts in geophysical fluid dynamics (such as Rick Salmon, who gave an excellent minicourse). Also, the strong input of geometric issues was very valuable to me.

2. What was your impression of the overall academic value of the workshop?

*Overall of high value. There was a broad enough range of talks for each day to contain something interesting, and many of the talks were of a high standard.

*Excellent value. It allowed people from all parts of the world to interact.

*Extremely nice people (which is not the case in all thematic communities of

mathematicians) of top international level. Just perfect to challenge future ideas and interactions.

*The workshop was at a generally high technical level and focussed on current research problems. There was genuine interaction between groups from diverse subject areas.

*Very good

*I think that the workshop was of highest academic quality, attracting the main specialists in the area of optimal transport.

*Excellent (very prominent researchers in their respective fields)

*Excellent!

*The impression was excellent, the quality and the originality of the talks was extremely high, the choice of the arguments was well balanced.

*It was extremely high. Although the days were quite long, the lectures were excellent, and the long breaks gave the opportunity for direct discussion between participants.

*I believe that the organizers did a great job in inviting a fantastic group of extremely active researchers in this area.

*Very valuable. Real connections were made between very applied people and pure mathematicians. This is something attempted at many workshops. Here it succeeded. This is a field -- for some reason -- where careful thinking about the applied issues has led to significant advances in pure math. (I hope the reverse is true as well, but won't speak to that!) This workshop will carry that forward.

*I thought the program was great. The speakers were good, if a bit eclectic.

*Personally, I felt the academic value was significant. Having read many papers in the build up to this conference, it was fantastic to meet the authors of many of these papers. In some cases, the discussions which followed on from meeting these individuals were very helpful with my research.

*Very high.

*An excellent conference – the fact that the organisers were able to attract big names from both optimal mass transfer and geophysical fluid dynamics is greatly to their credit. There was a good variety of talks, some introductory, some technical, and stimulating ideas were presented for both theoretical advances and further applications. It was good to see a number of younger academics attending; the poster session was an excellent opportunity for less experienced researchers to present their work.

* felt I learned quite a bit from this workshop. I really enjoyed learning more about the physics of GFD.

*Very high

*The workshop was highly interdisciplinary, and I think that participants found this of great value. There was a lot of informal interaction between groups with different expertise.

*The academic value of the workshop was excellent, with very renowned speakers, and the talks delivered were of very high quality.

*The workshop maintained throughout a very high academic level. More importantly, in this rather new field it is great to have such a broad/large conference in order to keep track of the direction research is taking. At the same time, a nontrivial effort is being made to engage new mathematicians in the area.

*The presentations were very nice at their own, but there was no clear coherence between the optimal transport theory and geophys.fluid dynamics.

It might be a good idea to include some time in the program for collaborations, to encourage linking transportation theory with geophys fluid dynamics

Because transportation theory and geophysical fluid dynamics are such different fields, the minicourses by Rick Salmon and Felix Otto were a good thing. It might be a good idea, though, to have some coherence between these two minicourses, so that the audience can explicitly see where transportation applications can be found in geophysical fluid dynamics.

*Very good.

*Very high level.

*The general standard of talks vas very high, both in mathematical terms and in quality of presentation.

*Excellent

3. What in your view are the key future research areas / directions in your field?

* My field is the calculus of variations which is related to but still fairly distinct from optimal mass transport - its closest relative among the topics of this workshop. So it's hard to give a straight answer to this question. I think there are ways for interaction between my field and some of the techniques of optimal mass transport which we saw at the workshop. These could be:

(a) use of the `right' cost functionals to make the dynamics of nucleation of martensite in austenite into a gradient flow, a la Otto-Kohn (this would be very tricky).

(b) the role of local minimizers in the Cullen-Norbury-Purser principle and in relation to the uniqueness of weak solutions of the semigeostrophic equations.

*Essential concepts in hydrodynamics and optimal transportation, applications.

*Well, too vast a question for a quick answer.
... It seems already important to improve our understanding of smooth optimal transport in several directions, like: Riemannian geometric interpretation of cost-curvature; curvature calculations for the Fischer information versus the Otto metric.

*Transportation problems in Riemannian geometry need further development, and there are many models in statistical mechanics which need new ideas. Invariant measures in infinitte dimensions is another topic which should be developed further. [Bourgain has made significant advances in the case of KdV flows, which were not treated at this meeting.]

*There's a lot of work to do in order to understand the models or create new ones.

*The geometry of the diffeomorphism groups, geometry of the geostrophic and related equations, the smoothness questions, global features for the transport on manifolds.

*Applications of ideas from Wasserstein spaces to fluid dynamics

*My field is numerical methods for geophysical fluid dynamics. Some of the key questions are concerned with what are the most important properties of the continuous governing equations that a numerical approximation should respect, particularly near the resolution limit. Exposing numerical modellers to rigorous analytical approaches to solving the governing equations can help move this area forward, and there were several presentations (oral and poster) at the workshop relevant to this theme.

*Applications of Optimal Transportation to nonlinear evolutionary PDE's and to the study of the geometry of metric measure spaces with Ricci curvature bounded from below.

*The great challenge in my field is the best use of computers to stud fluid dynamics Current practice follows an engineering approach, but I think that a more abstract approach offers the prospect of a more efficient and physically accurate result. This was the general theme of the meeting.

*Connections with mean field games, Hamilton-Jacobi equations and Aubry-Mather theory.

*I am in the early stages of my research career, and therefore do not feel I am experienced enough to comment on the future directions of the field.

*- the study of optimal transportation models with concentration and congestion
phenomena;

- the study of differential equations on spaces of probability measures;

- the links between optimal transportation theory and geometry.

*Clearly a key topic is to what extent results for optimal mass transfer on “nice” Riemannian manifolds extend to metric measure spaces. From a personal point of view, I am particularly interested in costs where the solution of the Monge-Kantorovich problem does not yield a solution of the Monge problem, and the consequences for generalised ideal fluid flow.

*I my view the key future research directions in my field is to try to adapt ideas from Optimal Transportation to nonlinear systems. In particular being able to exploit geodesic convexities in geometries other than Wasserstein.

*A direction close to the subject of the workshop: finite-time singularities in the GFD models

*My field is the geophysics area, and the key area is the application of rigorous mathematics in a way that is soundly physically based. The mixture of people at the workshop with a good geophysics background with rigorous analysts should have contributed to this. Analysis of the properties of asymptotic approximations to the governing equations is likely to be the most profitable area, and the mathematical methods presented at the workshop are highly

suitable for this task.

*This field (optimal transportation and applications) is developing fast, and this development seems to go faster and faster, as new people are getting into this area. Key directions are, in my opinion:

- continue to understand more about the links between geometry and optimal transportation, concerning regularity issues, links with Riemannian geometry and curvature. In this direction, the links with Ricci flows seem a very promising direction of research.
- improve the understanding of geophysical applications such as the semi-geostrophic equations. Most importantly, a uniqueness result for weak solutions would be a major achievement.

*I got the feeling that the research effort is taking a definite turn towards geometry. There seem to be deep connections between Mass Transportation and Symplectic Geometry yet to be understood.

*

- More applications of optimal transportation theory in GFD
- Continuation of designing sophisticated numerical schemes for GFD models
- Turbulence parameterization

*The interplay between geometry, fluid mechanics, inequalities, and optimal transportation.

*Further application of the mass transportation theory to diffusion equations in Riemannian manifolds.

Application to biological models (see for instance the talk of Slepcev)

*Use of transport equations in connection with vorticity.

*Optimal transport theory is very developed on its theoretical side, much less in its computational side. A strong effort should be made to get efficient large scale computational methods. On the theoretical side, I have suggested since 2000 to study the concept of optimal transportation of currents and I did some contributions. However, few people have shown interest in following this way.

4. Did the workshop help you to develop/sustain contacts likely to result in new research? [see footnote]

* Yes. I hope to work on the question of uniqueness of weak solutions to SG with Mike Cullen. I also met one of the principle British mathematicians who has agreed to help me with some of the technical issues involved with the project I mentioned above.

*I enjoyed very much talking to participants, some of whom I had not met for a very long time, or even never met.

*Contacts: yes; particularly with the Toronto people, but not only. New research: wait and see! In any case, I will be pleased to acknowledge stimulation by the **ICMS WORKSHOP** and let you know, once a paper is written.

*Yes, I had the opportunity to discuss topics with colleagues whom I would rarely meet otherwise. These provided updates on recent progress on research in various topics.

*Yes. I met some people with similar interests to mine that I didn't know before. If not a collaboration, at least I met people I can talk to about similar problems. On the other hand, I learnt about past works that relate to mine that I was not aware of before.

*Yes, of course. I established several contacts which might lead to new research.

*Yes x 4

*Yes. I think this aspect is especially valuable for younger researchers.

*Yes, I discussed some possible new research directions with some of the participants. Optimal Transportation is a subject rapidly growing in this period, therefore workshop like this one are quite useful and stimulating.

*Most definitely. I renewed several valuable contacts and initiated several others. In fact, much of my time since returning has been spent emailing to colleagues at the meeting.

*Yes, during the meeting Robert McCann, Jose Carrillo, Dejan Slepcev and I got together to organize a meeting that will bring together a number of the participants here once more. I also had a chance to continue discussions about work in progress withJose Carillo, and possible work with a number of others.

*Given that this is the first conference that I have attended, it was more of a chance to acquaint myself with some of the greats in the field, rather than develop contacts which would result in new research area. However, as mentioned below, two contacts have been made from the Conference.

*Yes, a lot.

*Yes. Alexander Plakhov and I will soon be working together on a project (involving polar inclusions, a generalisation of polar factorisation of vector-valued mappings). It was a pleasure to meet Jonathan Bevan; I look forward to potential collaboration with him, Mike Cullen, and Ian Roulstone.

*It is hard to tell at this point.

*Yes. I had detailed discussions with two existing collaborators and one new collaborator which should lead to future significant results. I also had discussions with several others which may lead to new areas of work.

*As mentioned above, this was a unique opportunity to meet such a vast audience of specialists of the field, and I definitely used this occasion to maintain contacts, and also create new ones.

*Definitely. I spent some time talking to Misha Feldman, which is always quite useful. I also spent a lot of time talking to Geoffrey Burton and to Robert McCann, which has proven useful and productive. It is possible that the interaction with Luigi Ambrosio could lead to new research.

*Yes, I had some very good input from other researchers inspiring for new directions in my own research.

*Yes... more on the sustaining for me, than the developing side, since I was swamped with collaborators at this workshop.

*I was able to meet with overseas collaborators whom I do not have many opportunities to see, and discuss ongoing collaboration.

*Certainly the workshop suggested to me new ideas that might evolve as new researches.

5. Did the programme result in new ideas or the acquisition of new techniques or methods?

* Yes. I wasn't aware of the Kohn-Otto methods which I hope to apply in some way to my own work.

*Subtle and elaborate new techniques, which would have been difficult to understand by reading research papers

*Yes. I never got such a challenging overview of the different aspects of the geometry of diffeomorphisms on a manifold, say on the sphere. From now on, I will certainly take better in account the fluid dynamical approach, viewing a diffeomorphism as the time 1 position of a fluid moving on the manifold - and what is the best motion?

Possibly, in the long run, it could inspire me for new methods in my own research.

*Yes, I was not previously aware of several new techniques, such as were presented in lectures by Carlen and others.

*Not really in my case, but I did learn about new open problems I can work on.

*Yes, there were new ideas and techniques floating around. In particular, the relation to Ricci flows and non-holonomic geometry.

*Yes x 4

*The programme gave me new ways (geometry, optimal transportation) of thinking about ideas already familiar to me (geophysical fluid dynamics). I expect these to influence my future research.

*Yes. I returned refreshed and with several new ideas.

*Yes. I found the minicourses quite helpful, as well as many of the presentations. To name just a few: Yann Brenier, Robert McCann and Felix Otto.

*I learned both some new problems and some new techniques. Now the issue is to see what I can do with them!

*Following on from discussions with Yann Brenier, I have got an idea of some future directions. Furthermore, John Thuburn suggested some solutions to problems outlined in the poster I gave in the poster session.

*It was very interesting to meet people working in geophysical fluid dynamics and to see how optimal transportation can be used in this field.

*Rather than new methods/techniques, it was clarification and explanation of recent results. For example Gregoire Loeper's excellent presentation allowed me to appreciate the significance of the regularity results of Trudinger and co-workers. Felix Otto and Robert McCann were similarly informative about gradient flows and mass transfer on Riemannian manifolds respectively.

*I really liked the way Otto exploited Wasserstein geometry in his work. I think his work is definitely original and I really enjoyed listening to his talk.

*I was certainly made aware of several new and relevant results.

*Yes, I learned new things in many talks (for example the lectures given by Felix Otto).

*To a certain extent. I acquired a better understanding of the physical interpretation of the semigeostrophic equations through the talks of Rick Salmon and Robert Douglas. I became familiar with the new result on regularity of solutions of Monge-Ampere type equations. Additionally, I enjoyed Luigi Ambrosio's talk on generalized solutions of the Euler equations and his striking new result on the regularity of the generalized pressure.

*Yes, how optimal transportation can be applied in GFD

*Yes, particularly for me, a better understanding of the geophysical perspective on fluid mechanics.

*Too soon to say

*The geometric talks were very informative to me (Khesin, McCann, in particular)

6. Have you any comments on the facilities (at the workshop venue) and/or accommodation or on the administrative arrangements?

*Accomm was great. Admin arrangements worked fine. Wireless access and access to University Library were most welcome too. WS dinner was good.

*Excellent facilities and accommodation, beautiful surroundings, helpful hosts.

*The facilities at the David Hume Tower and the accommodation at Pollock Halls were both excellent. The administrative arrangements were brief and efficient, run by extremely kind administrators who took perfect care of us. I would accept willingly participating on a similar basis to another such conference !

*Everything seemed to be well organized and the staff were helpful. Pollock Halls is a perfectly good residence for this type of meeting.

*Quite good.

*The facilities were extremely good and the organization was perfect.

*Facilities and administration were very good; no comment on accommodation as I was staying with a local colleague (for work reasons)

*These were fine.

*The organization was perfect.

*The running of the workshop was exemplary. Every request was satisfied, and the staff were extremely friendly and helpful. I am particularly grateful to Ms. Burton for her help, and for the opportunity to visit Maxwell's house at a busy time.

*Everything went pretty smoothly.

*The conference was very well run in every way. The venue was fully conducive to the aims of the workshop, and everything ran very smoothly.

*The facilities were fantastic. There were no problems at all. The accommodation was far nicer than I thought it would be, bearing in mind it was student accommodation. I wouldn't have minded such accommodation when I was a student! As for the administrative team; their hard work made the conference run smoothly. I must thank Irene, Morag and Johanna for their hard work.

*The accommodation at Pollock Halls was very good and the administration very efficient.

*ICMS Staff were helpful, efficient, and courteous at all times. The administrative arrangements were very good. Lecture rooms were very suitable. The accommodation was fine.

*Facilities were fine, although having a common bathroom was a bit of a hassle.

*Everything was perfect (the reimbursement of travel expenses hopefully will not be too long)

*The facilities at the workshop venue were very good. The location was also convenient for the City Centre. The administrative support from the ICMS was excellent. The accommodation was in a very pleasantly laid out campus area and of a good standard. The only issue was the noise from other large groups, sometimes late at night.

*The housing was excellent as far as I am concerned, and the administrative arrangements as well.

The only negative aspect of the Workshop venue is that it closed soon after the last talks, so the participants could not use blackboards, tables and chairs, etc, to continue discussing. It would be nice if there was some space to congregate and discuss mathematics long after the last talks. Otherwise the facilities were very good.

*Accommodation was great.
Facilities were good, no comments.

*Very good.

*Good facilities (Wireless, etc.).

Not good: the impossibility to pay the registration fees and accommodation by direct credit transfer.

*All very good

*Excellent organization and accommodation

7. Have you any other comments?

*When do you plan the next meeting? (please, let me know far ahead) Maybe somewhat earlier in July would be better, if room is available at Pollock Halls.

I would like to call the organizer's attention to the name of Emile Reynoldovitch ROZENDORN (at MGU, Moscow State Univ.); he is an interesting author in

connection with the fields of the workshop; he wrote papers on 2D Monge-Ampere equations on the sphere related to meteorology, over 20 years ago. If still active (he seems to be so), it might be a good idea to invite him next time.

*In many grant applications the phrase 'interdisciplinary research' is debased; whereas in this meeting, there was genuine interdisciplinary work from researchers who use transportation techniques in diverse areas. Normally, there is not much interaction between workers in theoretical fluid mechanics and differential geometry, and this meeting brought them together profitably.

*As a suggestion, in my opinion it would be somewhat more convenient to have a conference in a room, which has seats for say twice as many people as participants. This would make the sitting less "crowded" and more comfortable. Otherwise, the room was perfect. Thank you very much!

*Just a sincere thanks to everyone involved.

*Thank you for hosting this event, and for inviting me!

*The program was ambitious.

*I would like to thank all for my invitation, and the chance to give a poster. It has really helped my research, and I feel it will hold me in good stead for years to come.

*The topic of optimal mass transportation is having a large diffusion in the mathematical community, mainly because its flexibility to be applied to problems in economics, statistics, image matching, cosmology and geophysics, and also for its strong links with geometry and partial differential equations. I hope that ICMS, after the workshops of 2005 and 2007, will continue to help the development of this field.

*Particular thanks to Mike Cullen who did a great job as conference organiser.

*Wonderful workshop. I truly enjoyed it. It was definitely a huge benefit for the workshop that it took place in the beautiful city of Edinburgh.

*Weather was awful - could be prepared better

*Thank you very much to all the people that made this event possible!

*This is the third ICMS Workshop I have attended and, as with the previous two, it was very well organized and sustained a high academic level. I applaud these initiatives and hope they will continue taking place, as they are very valuable. In particular I wish to mention that the talk I gave in this Workshop was work which resulted directly from having attended the Optimal Transportation Workshop of 2005. The paper is still being written up for publication but it will surely acknowledge that meeting.

*I thank the organisers very much for inviting me and giving the opportunity to speak at the workshop. It was a very valuable experience for me. I learnt a lot, and met interesting people.

*Analysis applied to hydrodynamics would go well with the topics of this workshop

*I am very supportive of the ICMS, that I visited several times: exciting topics, excellent organization (one of the very best among

similar institutions), great location. In my opinion, ICMS is definitely world-class and deserves a long life!

Footnote:
ICMS asks that participants acknowledge the part played by the workshop in any papers resulting from research stimulated by participation in the meeting. Please let ICMS staff know of this acknowledgement and of any links that should be made from our webpage report on the workshop.