Probabilistic Limit Laws For Dynamical Systems

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Detailed Report:

The meeting had 35 participants; 18 from the UK, 9 from Europe and 8 from the US. There were 19 talks concerning limit laws for physical systems or mathematical models and the related mathematical or probabilistic techniques.

The comments we received were very positive, a representative example being:

> "Overall the standard of talks was excellent. Would be hard/unfair to pick out highlights. It was very noticeable that the discussions after seminars were open and not in cliques"

Other comments included:

> "The workshop was very focused and it has been an excellent opportunity to review many of the different tools used to understand limit laws of dynamical systems as well as the applications which can be made of them"

> "The workshop was very effective and allowed for unusually deep discussions between scholars who share common interests with different points of view and techniques.

In the opinion of the organisers here are some of the highlights of the meeting.

Highlights.

Sebastian Gouëzel gave an excellent talk on the statistical properties of stadium billiards. He divided Hölder observables into two classes, each requiring different scaling factors to ensure a Gaussian limit law. For typical observations the scaling is the nonstandard one $(n \log n)^{-\frac{1}{2}}$ whereas the standard scaling $n^{-\frac{1}{2}}$ applies for the remaining (atypical) observations. This was an unexpected and elegant result, and many participants cited this talk as a highlight of the conference. In a similar vein,

Domokos Szasz gave results on the statistics of the infinite horizon Lorentz system. For typical Hölder observables, again the nonstandard scaling $(n \log n)^{-\frac{1}{2}}$ leads to a Gaussian limit law.

Omri Sarig outlined his recent work on critical exponents for dynamical systems — part of an ongoing project to understand phase transitions. Previously, Sarig developed a thermodynamic formalism for countable Markov shifts. In his talk he described the relation between the expansion of the pressure function of an observable and the limit law the observable obeys.

Marta Tyran-Kaminska gave greatly improved results on the relation between the rate of decay of the norm of the Perron-Frobenius operator and the existence of a functional version of the central limit theorem. The proof used recent results from probability, specifically by Maxwell & Woodroofe, that apparently were not previously known within the dynamical systems community. This generated much discussion and should allow several other limit theorem results to be sharpened.

Other speakers gave very interesting results on rates of decay of correlations. Stephane le Borgne gave examples of skew extensions of Anosov systems which have very slow rates of decay of correlations for Hölder observables and non-Gaussian limit laws where the scaling factor is $n^{-\frac{3}{4}}$. Mark Holland spoke about the statistical properties of equivariant observations on compact group extensions (a setup which arises naturally when modelling systems with symmetry) using renewal theory ideas of Gouëzel and Sarig. Michael Field discussed stability and rates of mixing in hyperbolic flows. He also gave reasons why new techniques may be needed to answer the question of stability of exponential mixing. Carlangelo Liverani spoke of his work on developing a two-dimensional model for intermittency. The one-dimensional Pomeau-Manneville maps have played a major role as models in gaining insights into rates of decay of correlation and limit theorems and the development of a corresponding higher dimensional model would be an accomplishment.

Jens Marklof described limit theorems for the distribution of lattice points in circular and elliptic strips, and in annuli parametrised by a radius R approaching infinity. In the case of $\alpha \mapsto n^2\alpha \bmod 1$, the scaling is the standard one $n^{-\frac{1}{2}}$ but the distributional limit is not Gaussian. Indeed, the limit distribution is expressed in terms of an almost modular function, and falls outside the classical stable laws.

François Ledrappier and Roland Zweimüller talked about problems associated with infinite ergodic theory. Ledrappier considered a class of horocycle flows with no finite invariant measures, and showed that amongst the infinitely many ergodic invariant Radon measures, there is a unique one (the volume measure) satisfying a generalised strong law of large numbers. Zweimüller discussed distributional limit laws for a variety of one-dimensional maps with σ-finite ergodic measures. Both of these talks relied heavily on ideas of Jon Aaronson.

Manfred Denker described techniques, using the Chen-Stein method, to establish Poisson limit laws for certain hyperbolic systems.

Oscar Bandtlow discussed results on expanding maps with infinite branches, using decay of derivatives (instead of the more usual bounded distortion) to prove existence of an absolutely continuous invariant measure.

Two speakers discussed work-in-progress on physical systems for which a statistical understanding needs to be developed. Robert MacKay posed an interesting set of problems concerned with the dynamics of biomolecular motors related to obtaining a canonical ensemble from microcanonical ensemble. The hope is that recent developments on the ergodic theory of partially hyperbolic dynamical systems should allow a rigorous understanding. Renato Feres discussed work on limit laws in a billiard system motivated by an engineering problem: to determine the fine structure of the walls of a long pipe from the exit time statistics of particles traversing the pipe.

Involvement of participants and new collaborations.

The amount of informal, excited discussion that followed each talk was very rewarding to observe. The meeting will lead to new collaborations and research. Some of the comments by participants along these lines included:

> " Recently there has been a flow of ideas from probability theory into ergodic theory settings (esp. Aaronson, Denker etc) and this programme was intended to further facilitate the acquisition of ideas. I think it succeeded."
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> "Plenty of new ideas and methods for developing the theory, proving results".
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> "I learnt about interesting examples and limit theorems of non-hyperbolic dynamic systems..."
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> "Most certainly — our joint paper will acknowledge."
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> "It is likely that some new results discussed will be published."
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> "Although I didn't start an immediate project with any of the participants, several discussions certainly helped me in my research."