

Report on the ICMS workshop Algebraic K- and L-theory of Infinite Groups

Edinburgh, 27th June - 1st July, 2005

Ian Hambleton, Andrew Ranicki 6th September, 2005

The workshop had 35 participants from 12 countries: Belarus (1), Canada (1), France (1), Germany (7), Italy (1), Mexico (1), Netherlands (1), Norway (1), Spain (1), Switzerland (1), United Kingdom (6), United States (13). The scientific programme consisted of 19 formal lectures, which stimulated many informal discussions. We begin by reporting on the contributions of the main invited speakers.

The opening lecture by Wolfgang Lück outlined the current state of our knowledge of the computational methods of the algebraic K - and L -groups of infinite groups under the assumption that the Baum-Connes Conjecture or the Farrell-Jones Conjecture holds. Rationally a satisfactory general answer is given by equivariant Chern characters in terms of the homology of the centralizers of the finite cyclic subgroups.

Tom Farrell is one of the authors of the celebrated Farrell-Jones isomorphism conjectures, which set out a grand scheme for computing both the K - and L -groups of infinite groups. His joint research with Lowell Jones over the last 25 years used a variety of differential geometric and analytic techniques to verify these conjectures in many cases. In the afternoon of the opening day Farrell gave a lecture outlining some of the important analytic tools used in geometry, such as the geometry of harmonic maps, the Ricci flow and Teichmüller spaces.

Frank Quinn is a pioneer of controlled topology and the assembly map, which are central to both the statement and the proofs of the Farrell-Jones conjecture. In his lecture he outlined his recently completed controlled higher algebraic K -theory, and formulated a new conjecture on hyperbolic induction in the K -theory of infinite groups.

Erik Pedersen has made fundamental contributions to the controlled algebra used in the computation of the K - and L -groups of infinite groups. In his lecture Pedersen outlined a new method of attack on the problem of extending a solution of the Farrell-Jones isomorphism to proper subgroups. In fact, this is joint work with his former Ph.D. student David Rosenthal, who was also a participant at the meeting.

Jim Davis talked about a striking new theorem computing the L -theory of $PSL_2(\mathbb{Z})$, using the formulation of squeezing in L -theory, the proof of the isomorphism conjecture for crystallographic groups, and applications to equivariant rigidity. The theorem has interesting applications to connected sums of manifolds.

Martin Bridson is a world leader in geometric group theory. One of the purposes of the meeting was to expose topologists to the current thinking in this subject. In his talk Bridson considered the large scale geometry of mapping class groups and automorphism groups of free groups. The

structure and compactification of the classifying spaces EG of such groups G is an essential ingredient of the verification of the Farrell-Jones conjectures.

In addition to the main invited lectures there was a variety of talks selected by the organizers from the topics submitted by the participants. Aderemi Kuku spoke on the higher K -theory of group rings of virtually infinite cyclic groups, an essential ingredient for the Farrell-Jones conjecture in K -theory. This subject was also taken up by Daniel Juan Pineda, who showed that higher algebraic K -theory of such groups cannot be computed only from the K -theory of finite groups. Björn Jahren spoke on manifolds homotopy equivalent to $RP^4 \# RP^4$. Yuri Muranov spoke on codimension 1 splitting problems for closed manifolds and their applications to the computation of algebraic L -groups.

There were also impressive talks by young researchers at the workshop. The talk by Stefan Friedl on his joint work with Peter Teichner was particularly noteworthy, in which L -theory computations were used to construct the first new examples of topologically slice knots since the work of Freedman in the 1980's. Jeremy Brookman and Qayum Khan spoke on different parts of their joint work with Jim Davis classifying up to homeomorphism all closed manifolds homotopy equivalent to $RP^n \# RP^n$. Diarmuid Crowley and Jörg Sixt spoke on different aspects of their joint work applying chain bundles and Kreck surgery to manifold classifications. Dirk Schütz spoke on his computation of the Whitehead group of the Novikov ring, which arises in the topology of closed differentiable 1-forms. Marco Varisco spoke on his joint work with Lück, Reich and Rognes using topological cyclic homology and the cyclotomic trace to detect elements in the rationalized higher algebraic K -theory of integral group rings. Frank Bihler spoke on noncommutative localization in L -theory. Laurent Bartholdi spoke on self-similarity and infinite discrete groups acting on trees.

The organizers also spoke: Ian Hambleton described approaches to computing the K -theory nilgroups, and Andrew Ranicki spoke on his joint work with the late Des Sheiham on the algebraic K - and L -theory of free groups. The meeting was notable for its many informal interactions, as is clear from the comments of the participants, which we excerpt below.

What, for you, was the highlight of the workshop?

Talking with experts, in particular, Tom Farrell, about very specific topics in algebraic K - and L -theory.

The talks I found most consistently interesting and informative were those offered by Farrell, Davis, Pedersen and Bridson. The other highlights were the numerous conversation opportunities I had during the lunch breaks.

Excellent scientific programme.

The friendly welcome and stimulating discussions at the organiser's garden party.

Intimate group of very nice people. Conference good size with good mathematics.

What was your impression of the overall academic value of the workshop?

Excellent. There has not been a focused conference on this topic for some time.

The overall standard of the talks was very high. For me, the workshop was an important opportunity to maintain research contacts and remain current.

Very high. It was a meeting of high international standard.

This was a very high level workshop where important new results were discussed. It will certainly lead the way towards the discovery of further results in this very important area of mathematics.

I think the academic value of the workshop was very high, in that it provided excellent, up-to-date information about research in the field of algebraic K- and L-theory, as it is conducted both in the US and Europe.

Very high: all the leading experts were present, and available for personal discussions.

Extremely useful, due to the high quality lectures.

Did the workshop help you to develop/sustain contacts likely to result in new research?

Yes. Tom Farrell and I discussed a project throughout the conference and the week after. Although his idea was promising, it was unsuccessful. However, we are discussing collaboration on an extensive project.

Yes. I have started a new project with a participant with whom I have not written a paper before.

Yes. Ian Hambleton made some very helpful suggestions for my main research project: classifying manifolds via Kreck's modified surgery. Jim Davis also clarified a crucial part of my plan to classify 7-manifolds.

Absolutely. I will be working on understanding the Nil terms that arose in so many talks this week.

Acknowledgements

The workshop was funded by a direct grant from ICMS/EPSRC, a Scheme 1 Grant from the London Mathematical Society, and by the European Union RTN Network HPRN-CT-2002-00287. We are most grateful for their support, which enabled the success of the workshop. Finally, we would like to thank the staff at ICMS for their superb organizational support of the meeting.