

## Report on the ICMS Workshop (11–15 September 2006)

### “Metric Entropy and Applications in Analysis, Learning Theory and Probability”

#### 1. Significant deviations from the original proposal.

There has been no significant deviation from the proposal. However, because of some cancellations (due to personal reasons) of learning theory participants we had to slightly modify the list of key speakers. But thanks to the last-minute participation of Alexey Chervonenkis, one of the founders of statistical learning theory, this did not weaken the learning theory aspect of the workshop.

Shortly before the workshop Bernd Carl, who should give the first talk, suffered a serious car accident and could not participate in the workshop. His talk was taken over by one of the organisers.

#### 2. Short description of the meeting.

The main aim of the workshop was to bring together active mathematicians from three different areas – analysis, learning theory, and probability theory – the link being their common interest in metric entropy. The concept of metric entropy plays a prominent role and has interesting applications in all three areas, but so far there has not been very much co-operation between the different research directions. The workshop provided a forum to intensify the existing and to establish new contacts, giving all participants the opportunity to learn from each other, to exchange ideas, new results and techniques, to discuss open problems and promising directions of future research.

The core of the workshop were the survey lectures on the use of entropy in analysis (theory of function spaces, interpolation theory), learning theory (statistical learning theory, machine learning) and probability (small deviation problems). Some other main talks were devoted to important open problems connected with metric entropy (interpolation of compactness by the complex method, duality of entropy numbers), or relevant new developments in the corresponding fields. In the short contributions recent results on more specific topics were presented.

#### 3. Comprehensive report of the meeting

**Participants.** The meeting had 30 participants from 7 countries, namely from UK (8), Germany (8), Spain (5), USA (4), Israel (2), Poland (2) and Russia (1). There was a good mix of participants from each of the three areas, as well as a considerable number of younger (9) and female (6) participants. For some younger participants it was the first (or second) conference they ever attended. The “key participants” were all leading experts in their fields.

**Structure of the workshop.** The scientific programme consisted of 13 main talks (50 min) and 12 shorter talks (30 min). In view of the interdisciplinary

character of the workshop we had asked some of the main speakers to give survey talks, explaining the use of metric entropy in their fields. We scheduled these lectures in the first days of the workshop, with the idea to provide in this way a common basis for discussions, and this indeed worked out very well. It created a very open and friendly atmosphere, and initiated an intensive and fruitful exchange between the participants representing the different areas.

Other main talks were devoted to new ideas for solving some of the outstanding open problems related to metric entropy (interpolation of compactness by the complex method, duality of entropy numbers), or to relevant new developments in the three areas covered by the workshop.

The shorter talks offered the opportunity to present more specific results. We had 12 such contributions, among them several given by young participants.

Moreover, there was enough time for informal discussions in smaller groups. In particular, on Thursday afternoon we had an extra session (with about ten participants) in which Shiri Artstein-Avidan continued her talk and explained some more details of her partial solution to the duality problem.

**Scientific highlights of the workshop.** Most participants – and the organisers absolutely share this judgement – have seen the highlights in

- the excellent quality of the talks and
- the inspiring interaction of mathematicians from different areas.

It was common opinion at the workshop and also expressed in many responses to the questionnaire that Shiri Artstein-Avidan's lecture was one of the highlights of the workshop. Further talks mentioned in more than one response as outstanding, are the ones by Ingo Steinwart, Hans Triebel, and Thomas Kühn.

Here are some more comments taken from the responses to the questionnaire:

"For me the highlight was the workshop itself, i.e., the idea to combine topics (and people) from functional analysis, stochastics and learning theory within such an inspiring manageable atmosphere."

"Excellent invited talks, nice atmosphere"

"There was a number of very interesting talks followed by stimulating discussions."

"Some of the talks were exceptionally good and introduced me to some questions and open problems I did not know about."

"The invited speakers constituted the top of the world's group of researchers working in these domains."

"Bringing together a good number (but not a too large number) of mathematicians which are interested in three different research areas"

"The highlight was in interaction with people from different areas."

"To learn what is learning theory and how it is related to entropy"

"The good opportunities to have informal talks with colleagues, the carefully selected survey lectures"

"The excellent quality of the talks"

"The highlight for me was the fact that this was a meeting of mathematicians working at different fields yet using similar techniques and methods. It demonstrated the power of metric entropy techniques."

**Academic value of the workshop.** In the organisers' opinion the academic level of the workshop was really excellent, and this was confirmed by all responses from participants to this point in the questionnaire. The academic level was characterised as "extremely high", "very high, far above the average standard", "really fine", "quite high, exactly as one expected". One participant wrote: "Extremely high; both founders of big research areas (including Hans Triebel and Aleksey Chervonenkis) and young promising researchers were present". Another wrote: "I attend many conferences each year and I have been to many research institutions. This is one of the best, and it is the best one I have been this year."

Several reasons were given for this positive evaluation. Very often it was mentioned the high standing of the main speakers, who were described as "outstanding specialists", "top researchers in all of the topics of the workshop", "competent, frank and direct with other people", "very good scientists with much enthusiasm for cooperation and exchange of ideas". Quite important was also the opportunity to learn from each other, due to the interdisciplinary character of the meeting. One participant wrote "I learned so much that it will take months for me to digest." Moreover, the open and inspiring atmosphere, and the intense discussions across traditional boundaries contributed essentially to the value of the workshop. Several participants pointed out that the workshop led to new interests and insights, and made new contacts possible.

**Key future research areas.** A large number of topics have been named, ranging from quite general themes to very specific problems. Here are some examples: applications of analysis, including metric entropy and geometry of Banach spaces, in learning theory; interplay of small deviation techniques with analytic applications and vice versa; multiscale analysis and its relations to function spaces, fractal analysis, and entropy; entropy and approximation numbers of operators which have practical applications; closer links between pattern recognition/learning theory, as well as fractal geometry/stochastics/functional analysis; behaviour of compactness under complex interpolation; duality of metric entropy of operators

These key future research areas have in common that they are concerned with interactions between analysis - stochastics - learning theory, one important link being metric entropy.

**Contact between participants.** As already mentioned above, there has been intense discussions during the workshop, existing contacts have been strengthened, and new ones established which may lead to future collaborations. Some colleagues made already concrete plans for visits and joint research projects.

**Organisation.** ICMS was an excellent venue for a meeting of this type and size. All participants enjoyed the pleasant atmosphere at India Street 14 and praised the truly perfect organisation and constant attention and kindness of all ICMS staff members. The organisers are very grateful for the (administrative and financial) support that made this workshop possible.