Random Dirichlet polynomials: metric entropy

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We study the supremum of some random Dirichlet polynomials, e.g.

$$D_N(t) = \sum_{n \in \mathcal{V}_\tau} \varepsilon_n n^{-\sigma - it},$$

where $0 \le \sigma < 1/2$, (ε_n) is a sequence of independent Rademacher (or standard Gaussian) random variables, and $\mathcal{V}_\tau = \{2 \le n \le N : P^+(n) \le p_\tau\}$, $P^+(n)$ being the largest prime divisor of n and p_τ the τ-th prime number. We obtain sharp upper and lower bounds for supremum expectation that extend the optimal estimate of Halász-Quéffelec

$$\mathbf{E} \sup_{t \in \mathbb{R}} \left| \sum_{n=2}^{N} \varepsilon_n n^{-\sigma - it} \right| \approx \frac{N^{1-\sigma}}{\log N}.$$

Our approach in proving these results is entirely based on methods of stochastic processes, in particular the metric entropy method. It provides new estimates in Rudin–Shapiro problem for Dirichlet polynomials.