Interpolation of entropy numbers and the measure of non-compactness

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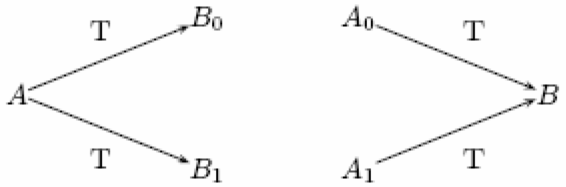
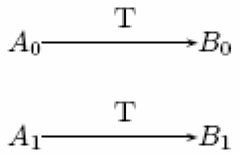
We use the real interpolation method as a conducting thread to describe the interpolation properties of the entropy numbers and of the measure of non-compactness. In a first stage we work with degenerated couples, that is to say, the operator satisfies diagrams of the type



In these cases, and for the main interpolation methods, the entropy numbers of T satisfy inequalities of the type

$$e_{n+m-1}(T : \overline{A}_{\theta,q} \longrightarrow \overline{B}_{\theta,q}) \leq C e_m(T_{A_0,B_0})^{1-\theta} e_n(T_{A_1,B_1})^{\theta}.$$

These inequalities produce their counterparts for the measure of non-compactness. However when T acts between two proper couples



we need different techniques to estimate the measure of non-compactness of the interpolated operator. Hereafter we describe two different approaches to this problem: In first place we study the contribution of Edmunds and Teixeira that use an approximation hypothesis on the couple (B_0, B_1), and that may be applied to a large family of interpolation methods. Next we will show a more general result for the real method with no additional hypothesis due to Cobos, Martínez and the present author. In both cases the estimate obtained is

$$\beta(T : \overline{A}_{\theta,q} \longrightarrow \overline{B}_{\theta,q}) \leq C \beta(T_{A_0,B_0})^{1-\theta} \beta(T_{A_1,B_1})^{\theta}.$$