Experimental investigation and applications of the modulational wave train

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Workshop on Rogue Waves :
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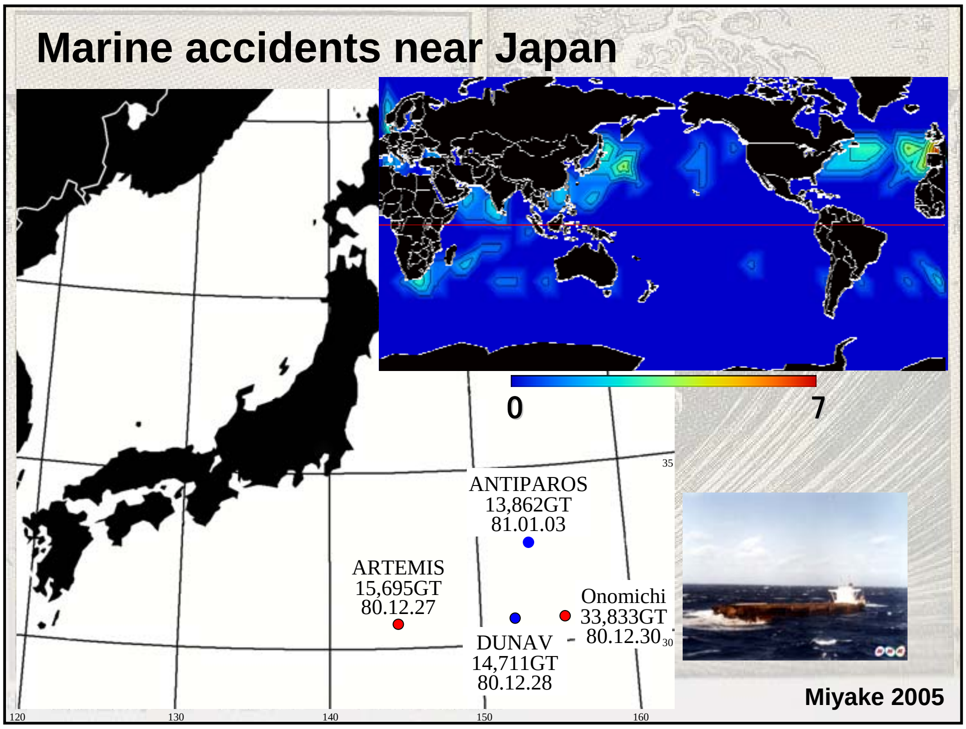
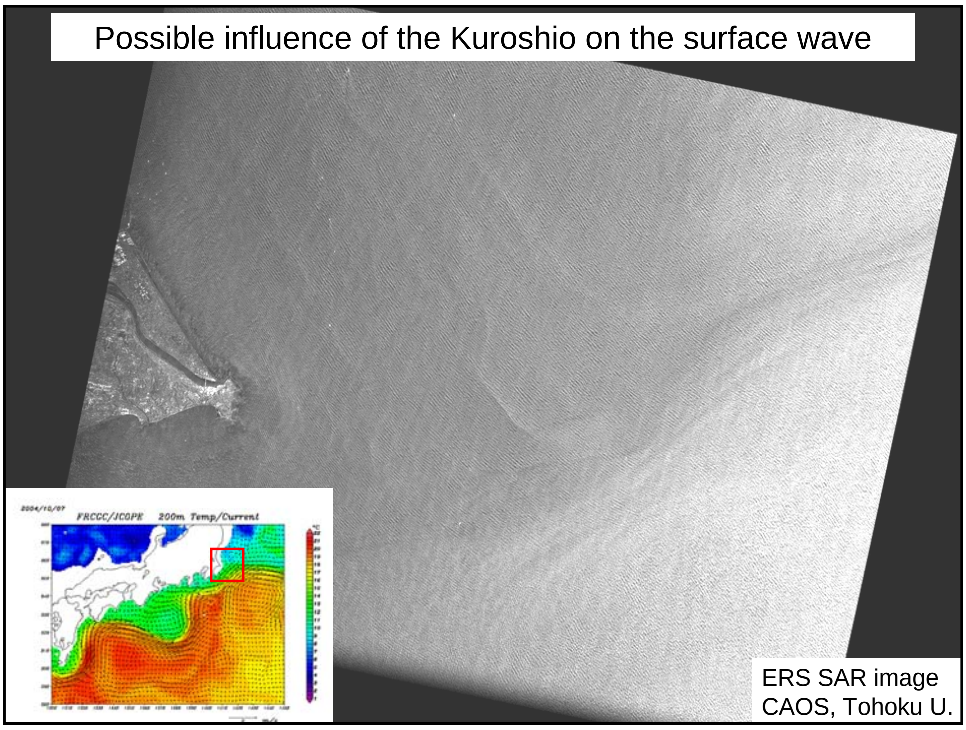
outline

1. **Introduction to the freak wave research at the U. of Tokyo**
2. **Highest waves in the modulated wave train – experimental evidence and comparison with weakly nonlinear theories**
3. **Summary**
4. ***Note on 3D wave generation in a tank***


Marine accidents near Japan
0
7
ANTIPAROS
13,862GT
81.01.03
ARTEMIS
15,695GT
80.12.27
Onomichi
33,833GT
80.12.30
DUNAV
14,711GT
80.12.28
35
30
120
130
140
150
160
Miyake 2005


Possible influence of the Kuroshio on the surface wave
ERS SAR image
CAOS, Tohoku U.

1. Freak wave research at the University of Tokyo with National Maritime Research Institute

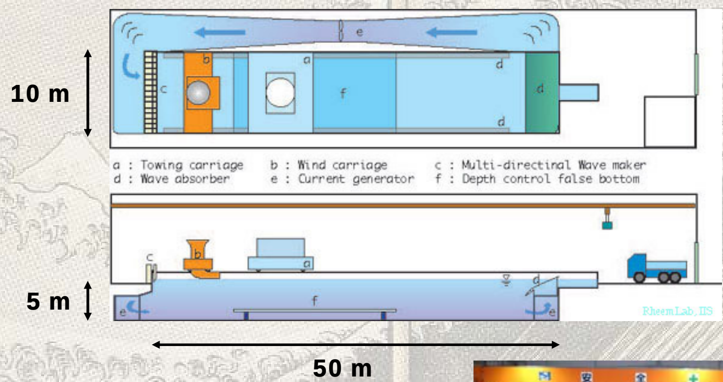
funded project (2004-2007)

- **Group I: Mechanisms, detection and numerical simulation of the freak wave**

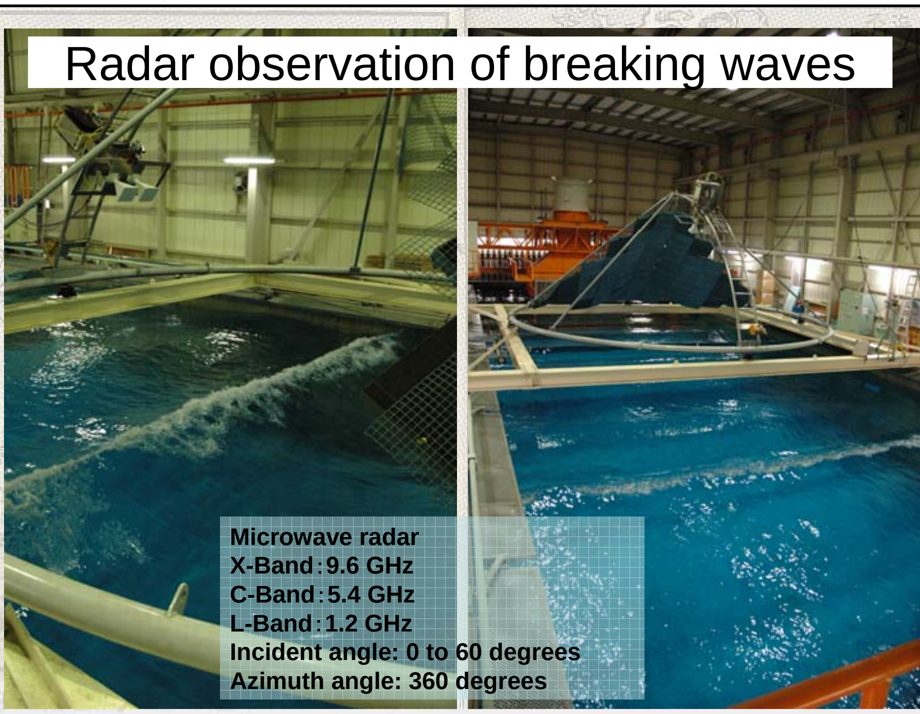
  (Tomita NMRI, Kinoshita, Waseda, Kawamura ,Rheem UT)
  - **Wave generation in tanks**
  - **Radar experiments in tanks**
  - **Numerical simulations**
- **Group II: Extreme wave loads on ships and design of an avoidance system**

  (Yuhara, Suzuki, Yamato, Kagemoto UT, Tanizawa NMRI, Miyake NK)
  - **Tank experiments**
  - **Numerical simulations**
  - **Avoidance system**
  - **Marine accident record analysis**

Ocean Engineering Tank
Institute of Industrial Science, U. of Tokyo



Radar observation of breaking waves



Wave generation in tank

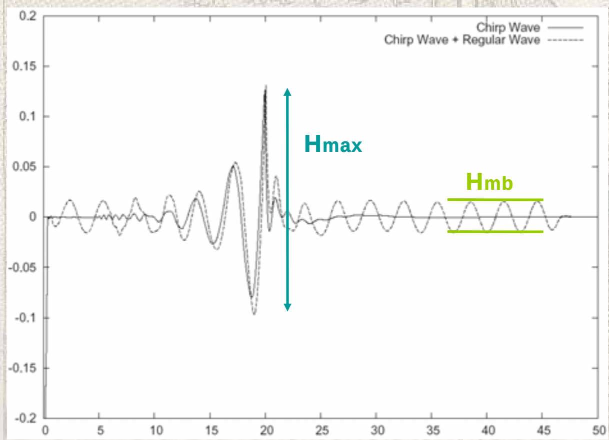
Extreme wave generation

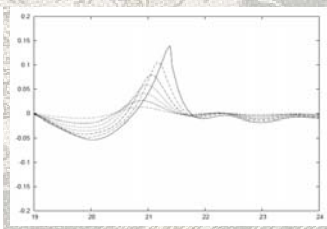
1. Linear focusing wave + regular wave
2. BF instability wave
3. Solitary type waves
   – Soliton, Ma-soliton

4. 3D waves (in progress)

Directional wave maker
32 plungers
0.5~5 s wave period

Combination of linear focusing wave and regular wave

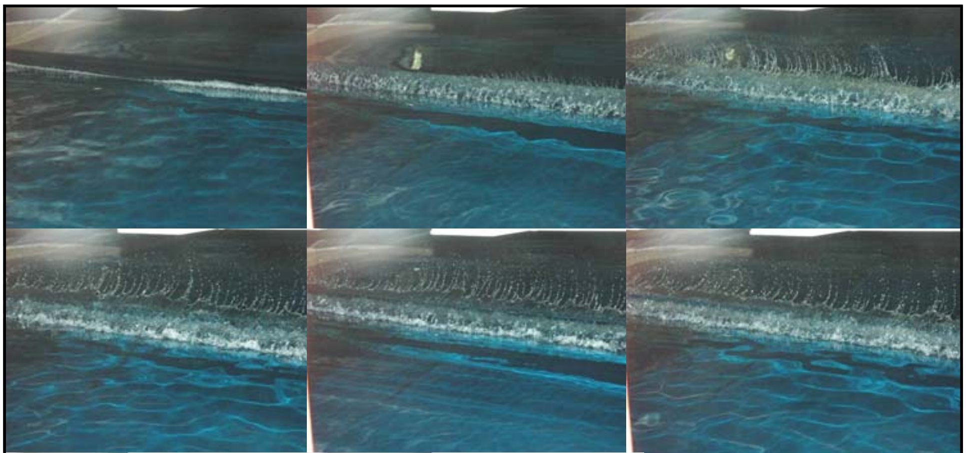




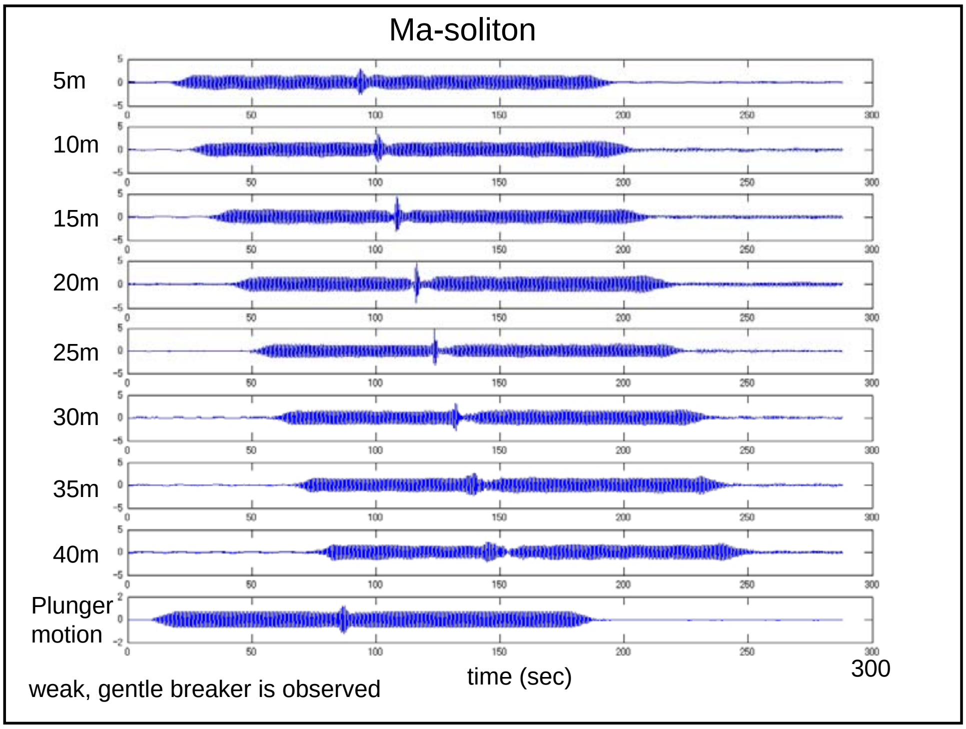
Variety of Hmax and Hmb combinations can be produced

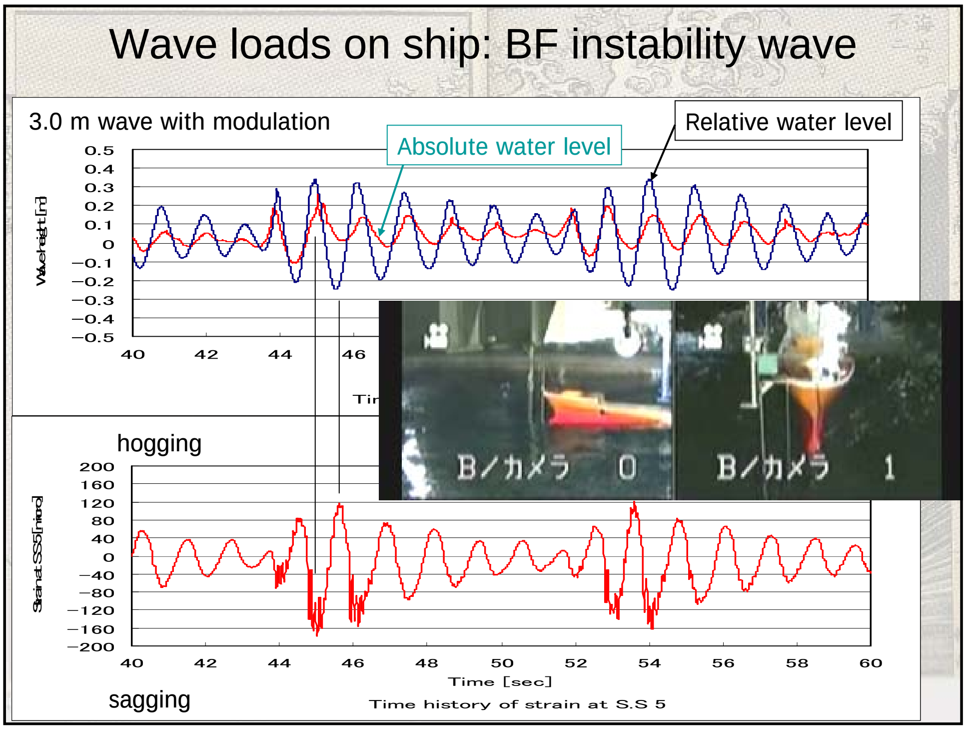
Focusing,
1m-5m waves
+
3m regular wave

Wave profiles are computed using fully nonlinear 2D numerical wave tank



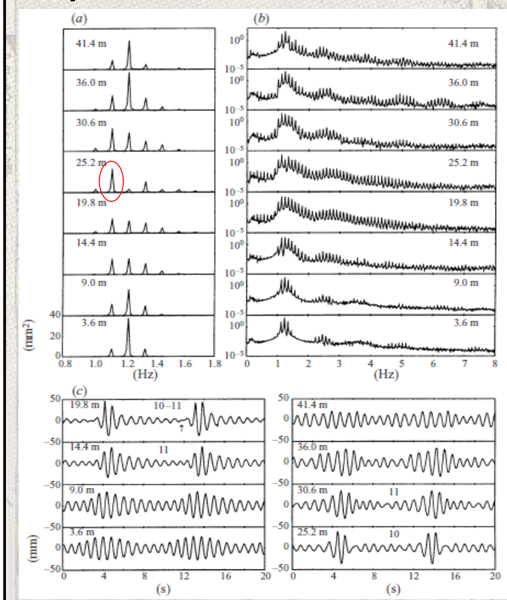
Energetic breaker of the focusing wave (1-5m) + small amplitude regular wave (3m)


Ma-soliton
5m
10m
15m
20m
25m
30m
35m
40m
Plunger motion
time (sec)
300
weak, gentle breaker is observed


Wave loads on ship: BF instability wave
3.0 m wave with modulation
Absolute water level
Relative water level
hogging
sagging
Time [sec]
Time history of strain at S.S 5
B/カメラ 0
B/カメラ 1

2. Highest waves in the modulated wave train – experimental evidence and comparison with weakly nonlinear theory

Experimental works on the modulational instability – review



Tulin & Waseda 1999

1967 Benjamin & Feir
"The disintegration of wave trains on deep water. Part 1. Theory"

1977 Lake & Yuen
Long-term evolution by seeded runs

1982 Melville
Spectral downshifting and breaking

1982, 1984 Su, Su et al
3D instability, wave pulse evolution

1986 Bliven, Huang & Long
influence of wind

1999 Tulin & Waseda
Long-term evolution over wide parameter ranges

1999 Waseda & Tulin
Initial growth rate; verification of theory

Generating Benjamin－Feir Wave train in a tank

Plunger motion

$\eta = a_c \cos(\omega_0 t) + b_+ \cos(\omega_+ t + \varphi_+) + b_- \cos(\omega_- t + \varphi_-)$

$a_0{}^2 = a_c{}^2 + b_+{}^2 + b_-{}^2$

$\omega_+ = \omega_0 + \Delta\omega$

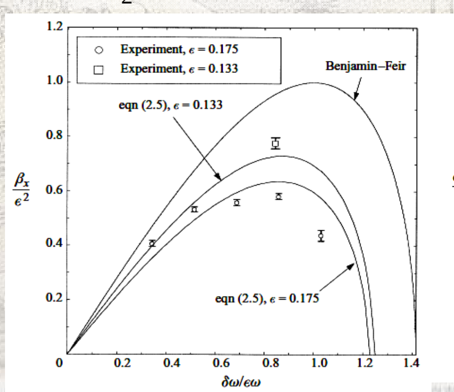
$\omega_- = \omega_0 - \Delta\omega$

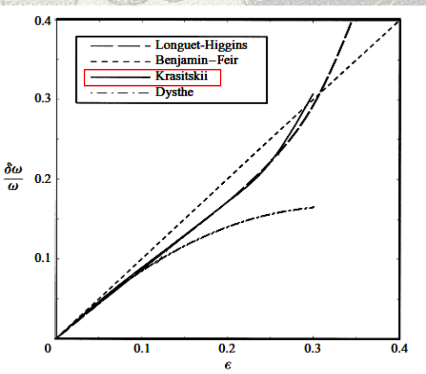
$\varphi_+ + \varphi_- = -\dfrac{\pi}{2}$

Control Parameter

$a_0 k_0$

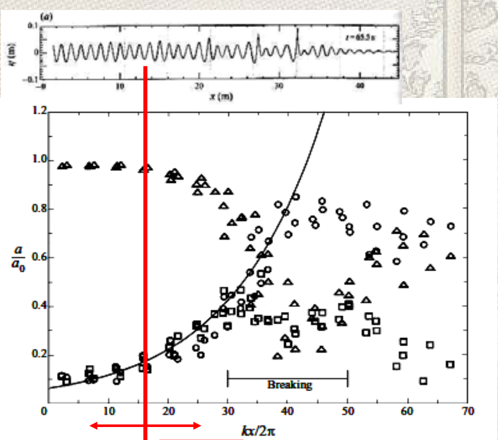
$\delta = \dfrac{\Delta\omega / \omega_0}{a_0 k_0}$





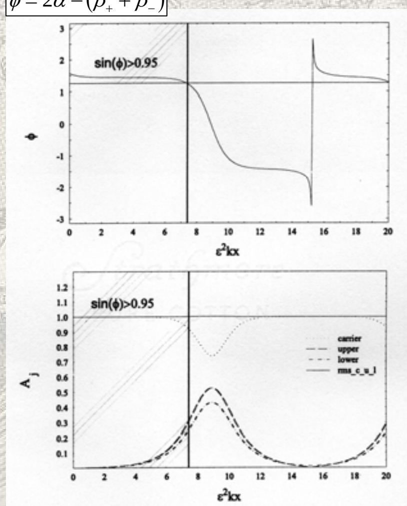
Waseda&Tulin JFM 1999

To observe long-term evolution of the Benjamin－Feir Wave train



$b_\pm / a_c$

$\phi = 2\alpha - (\beta_+ + \beta_-)$



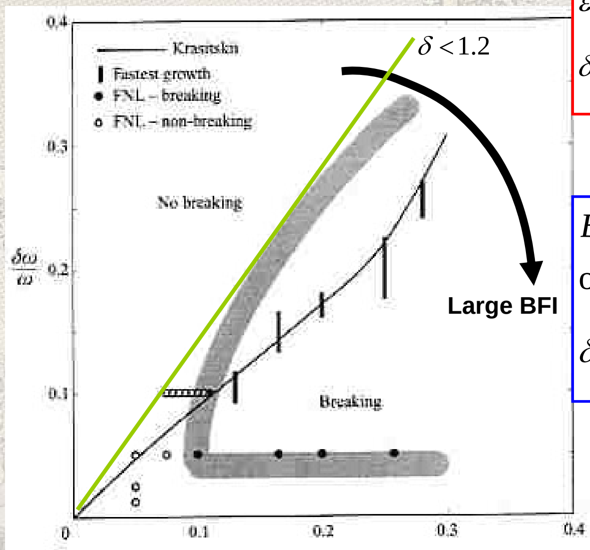
$\eta = a_c \cos(\omega_0 t) + b_+ \cos(\omega_+ t + \varphi_+) + b_- \cos(\omega_- t + \varphi_-)$

$a_0{}^2 = a_c{}^2 + b_+{}^2 + b_-{}^2$

$\varphi_+ + \varphi_- = -\dfrac{\pi}{2}$

Tulin & Waseda JFM 1999

Exploring the parameter space



Initial steepness

Tulin&Waseda JFM 1999

$\varepsilon = a_0 k_0$ Initial steepness

$\delta = \dfrac{\Delta\omega/\omega}{a_0 k_0}$ Band width

$$BFI = \sqrt{2}\varepsilon / (2\Delta\omega/\omega_0)$$

or

$$\delta = \frac{1}{\sqrt{2}}\frac{1}{BFI}$$

Instability δ < 1.2 or BFI > 0.6

BFI:Benjamin-Feir Index

Onorato et al. 2004

Janssen 2003

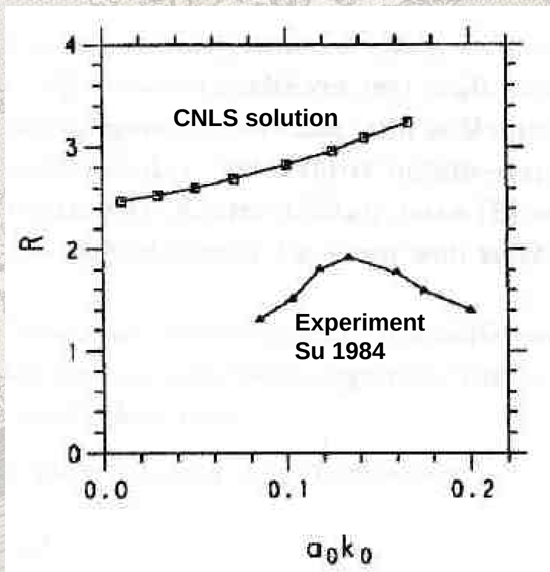
Is Freak/Rogue wave a breaking wave?

What is the role of breaking wave in determining the highest wave in a wave train?

Maximum amplification of waves in a modulated wave train

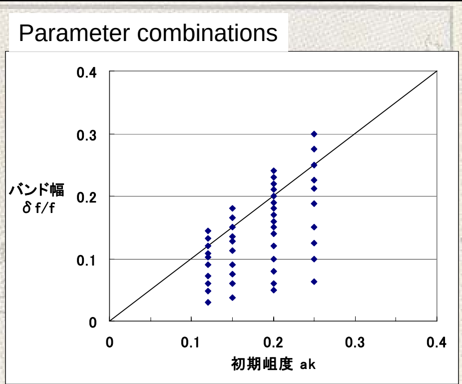
$R = H_{max}/2a$

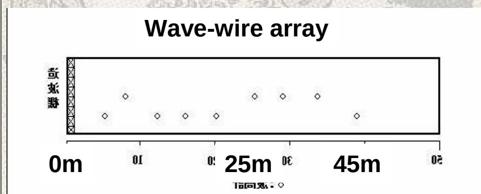
Enhancement Factor

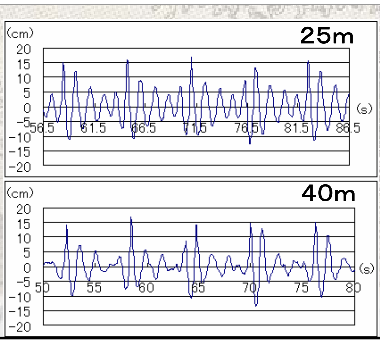


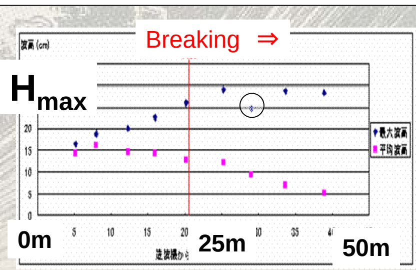
Tanaka 1990

Tank Experiment



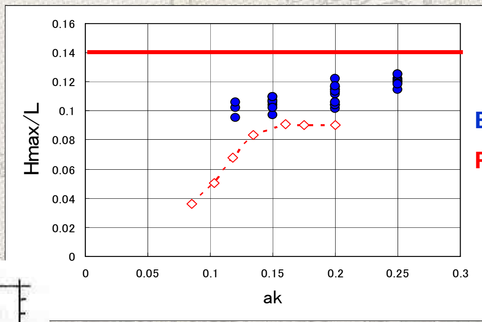






Comparison of observed maximum wave slopes from two experiments

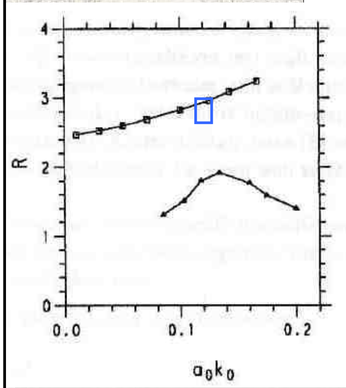
Maximum wave slope vs. initial steepness

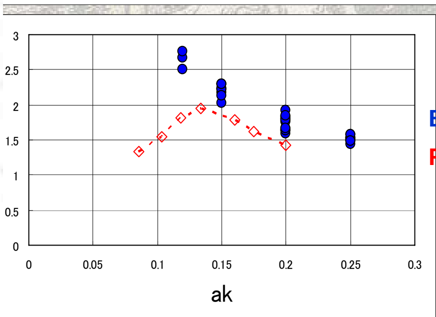


Stokes limiting wave

Blue: This study

Red: Su 1984



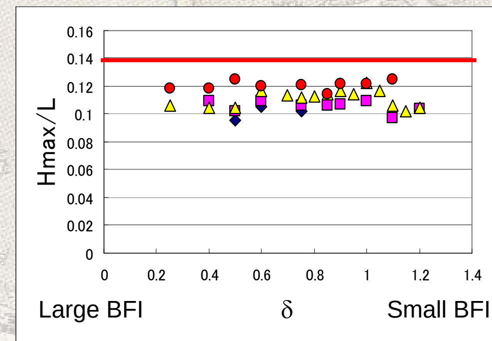


Blue: This study

Red: Su 1984

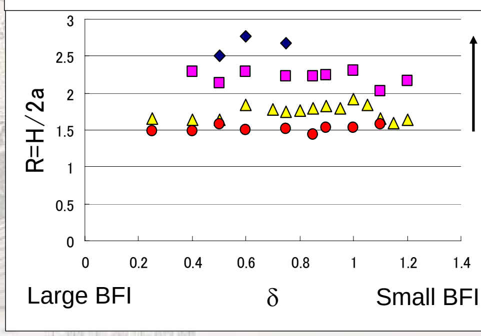
Observed maximum wave slopes from this experiment

Maximum wave slope vs. band width (1/BFI)



Stokes limiting wave

Enhancement factor vs. band width (1/BFI)



Small ak

Blue: ak = 0.12
Pink: ak = 0.15
Yellow: ak=0.20
Red: ak = 0.25

Numerical Solvers of the weakly nonlinear evolution equations

4 wave reduced equation (Krasitskii)

$$i\frac{\partial B_0}{\partial t} = \sum_{\mathbf{k}_0+\mathbf{k}_1=\mathbf{k}_2+\mathbf{k}_3} \tilde{V}^{(2)}_{0,1,2,3}\exp(i\Delta_{0+1-2-3}t)B_1{}^*B_2B_3$$

$$\sigma_0+\sigma_1-\sigma_2-\sigma_2=\Delta_{0+1-2-3}$$

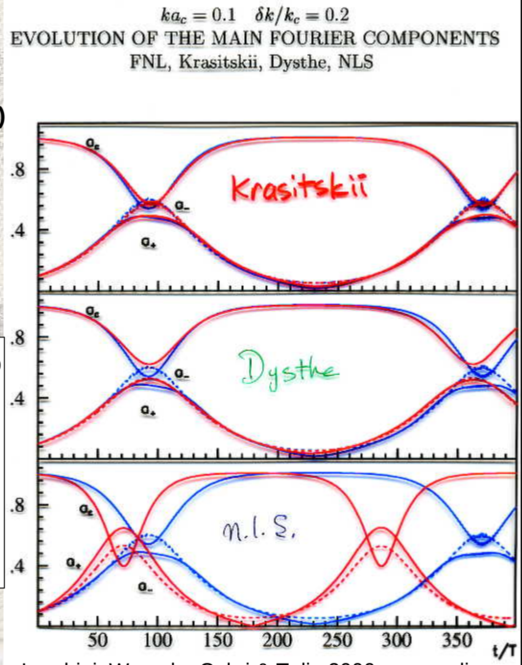
Dysthe's equation

$$\underbrace{\frac{\partial A}{\partial \eta}+i\gamma\frac{\partial^2 A}{\partial \xi^2}+i|A|^2A}_{\text{Cubic nonlinear Schrodinger equation}} \quad +8\varepsilon\gamma|A|^2\frac{\partial A}{\partial \xi}+4i\varepsilon\gamma A\left.\frac{\partial \phi}{\partial \xi}\right|_{z=0}=0$$

$$4\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial z^2}=0 \quad -\infty<z<0$$

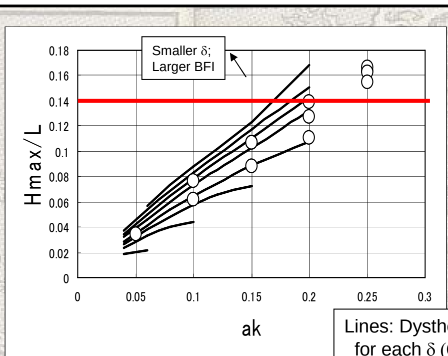
$$\frac{\partial\phi}{\partial z}=\frac{\partial}{\partial\xi}|A|^2 \quad z=0$$

$$\frac{\partial\phi}{\partial z}=0 \quad z=-\infty$$



Landrini, Waseda, Oshri & Tulin 2000, proceedings

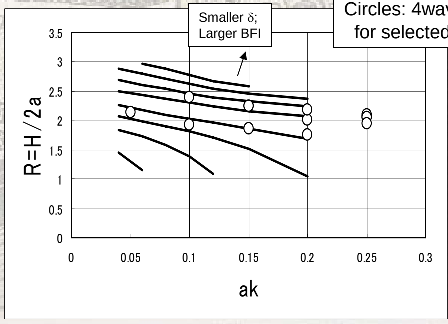
Maximum wave slope vs. initial steepness



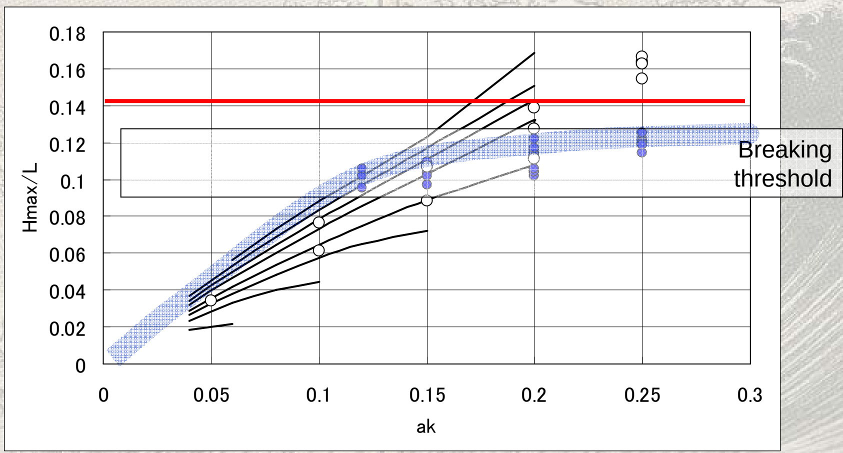
Stokes limiting wave

Lines: Dysthe's eqn,
for each δ (0.2,0.4,0.6,0.8,1.0,1.1,1.2)
Circles: 4wave reduced eqn
for selected δ

Enhancement factor vs. initial steepness

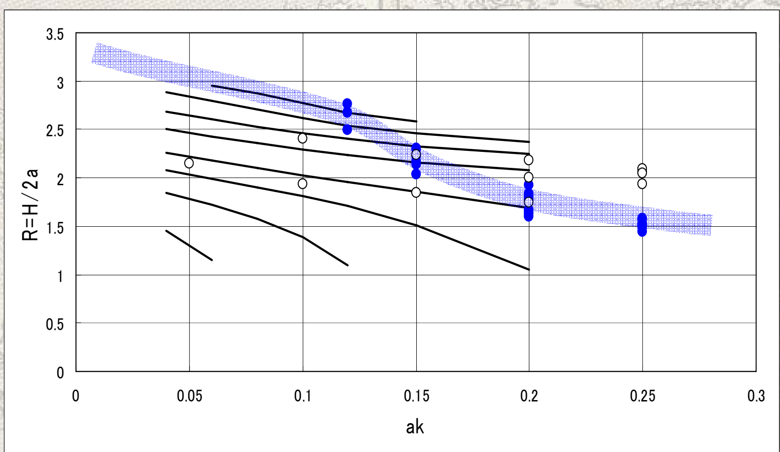


Comparison of experiment and weakly nonlinear solutions



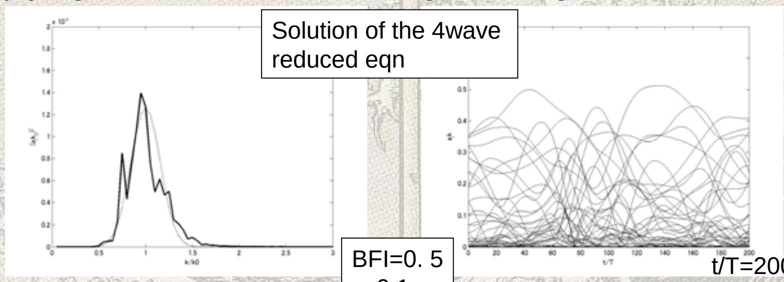
Lines: Dysthe's eqn
Open Circles: 4-wave reduce eqn
Blue Circles: Experiment

Comparison of experiment and weakly nonlinear solutions



Lines: Dysthe's eqn
Open Circles: 4-wave reduce eqn
Blue Circles: Experiment

Applying the result in determining breaking condition



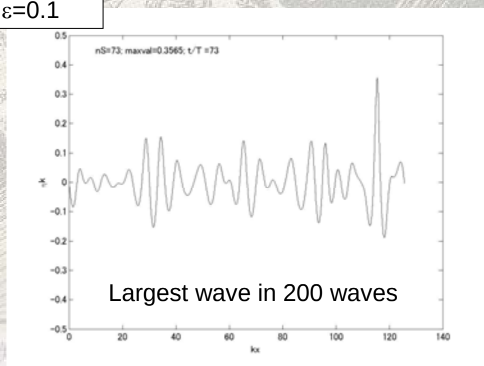
Gaussian spectrum

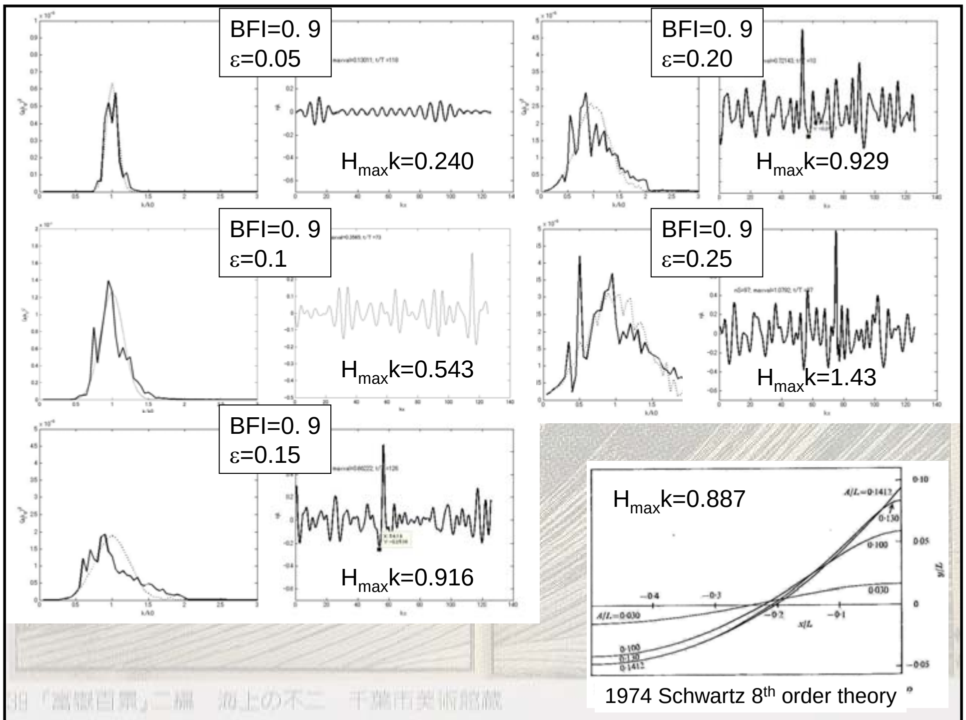
$$W_{oi} = \frac{BFI}{2\sqrt{\pi}} \exp\left(-\frac{BFI^2}{4\varepsilon^2} p_i^{\ 2}\right)\left(\frac{\Delta k}{k_0}\right)$$

Wave shape in observed variable

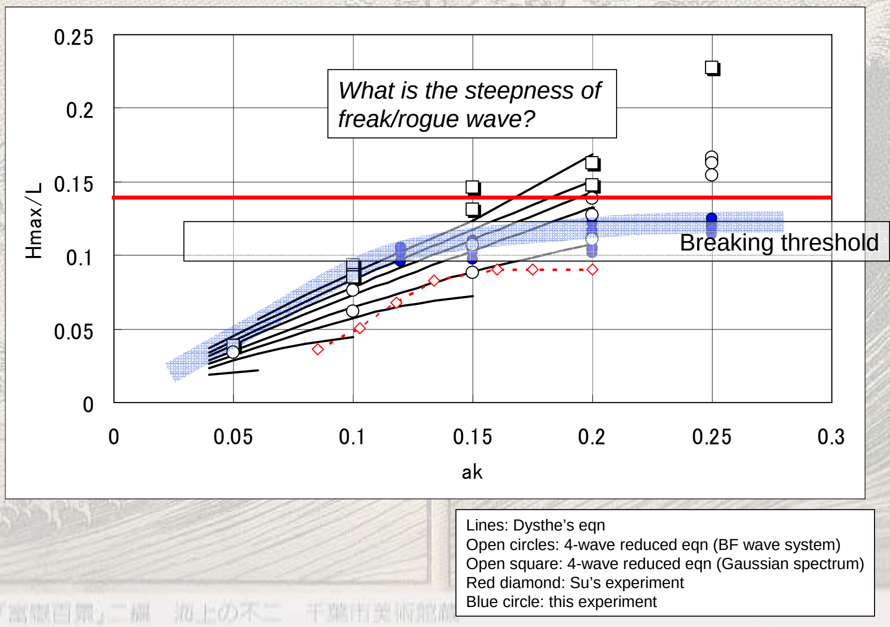
Postulate the canonical transformation from $a(k)$ to $b(k)$ in the form of integral-power series:

$$\begin{aligned} a_0 = b_0 &+ \int A^{(1)}_{0,1,2} b_1 b_2 \delta_{0-1-2} dk_{12} + \int A^{(2)}_{0,1,2} b_1^* b_2 \delta_{0+1-2} dk_{12} \\ &+ \int A^{(3)}_{0,1,2} b_1^* b_2^* \delta_{0+1+2} dk_{12} + \int B^{(1)}_{0,1,2,3} b_1 b_2 b_3 \delta_{0-1-2-3} dk_{123} \\ &+ \int B^{(2)}_{0,1,2,3} b_1^* b_2 b_3 \delta_{0+1-2-3} dk_{123} + \int B^{(3)}_{0,1,2,3} b_1^* b_2^* b_3 \delta_{0+1+2-3} dk_{123} \\ &+ \int B^{(4)}_{0,1,2,3} b_1^* b_2^* b_3^* \delta_{0+1+2+3} dk_{123} \end{aligned}$$



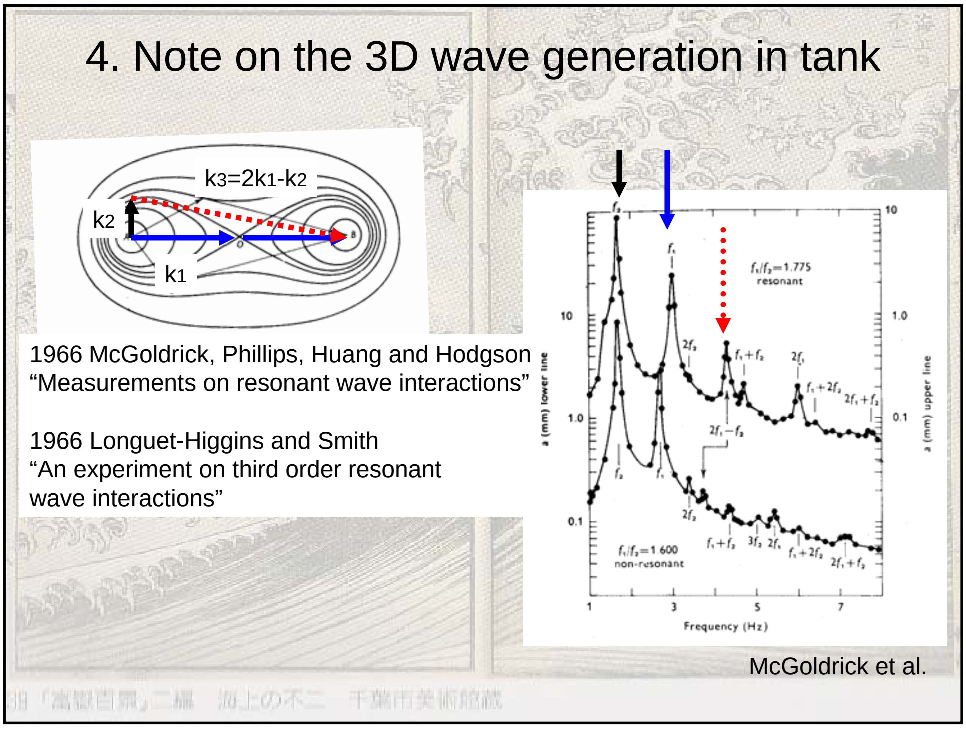


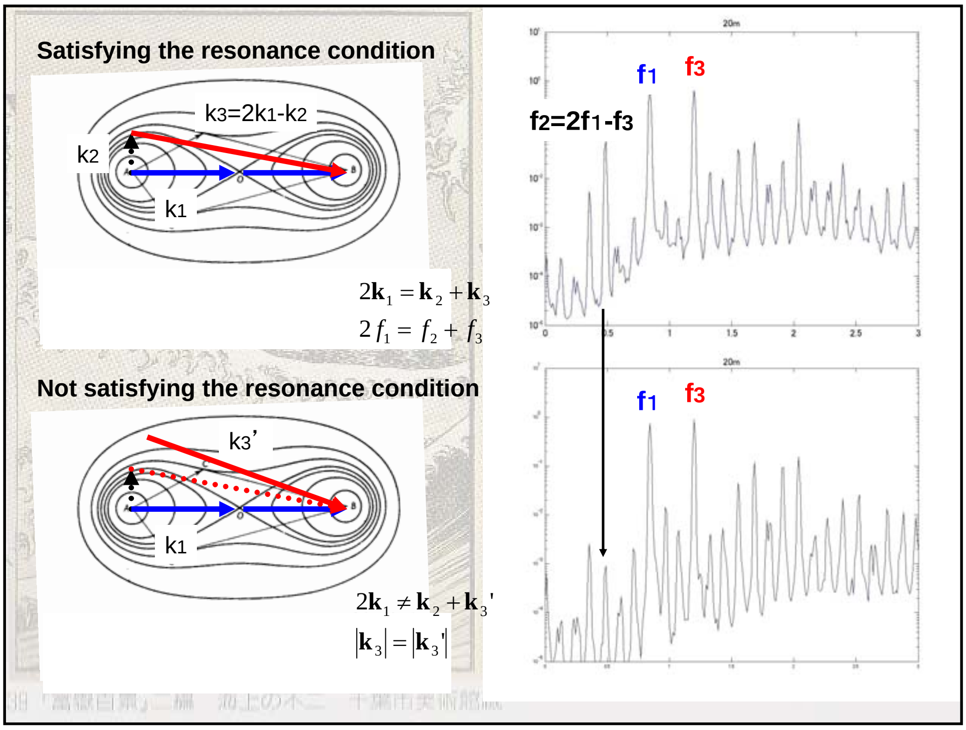
Summary Plot

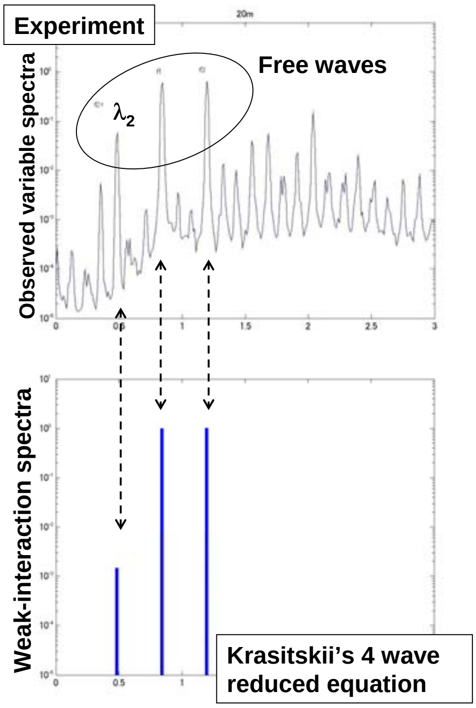


3. Summary

- Experiment on the BF-wave train covering a wide parameter space was conducted
- Comparison with weakly nonlinear theory indicates that the maximum wave height is limited due to wave breaking at higher steepness (ak>0.12 or so)
- Breaking wave steepness is smaller than the Stokes limiting wave
- Application of the findings from the study of BF wave train was demonstrated for the evolution of Gaussian wave spectra, suggesting that some waves predicted by weakly nonlinear theory are breaking in reality
- *Future: Further tank investigation is planned to verify the continuous spectrum cases including directionality*


4. Note on the 3D wave generation in tank
k3=2k1-k2
k2
k1
1966 McGoldrick, Phillips, Huang and Hodgson
“Measurements on resonant wave interactions”
1966 Longuet-Higgins and Smith
“An experiment on third order resonant
wave interactions”
a (mm) lower line
a (mm) upper line
Frequency (Hz)
McGoldrick et al.


Satisfying the resonance condition
k3=2k1-k2
k2
k1
$2\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$
$2f_1 = f_2 + f_3$
Not satisfying the resonance condition
k3’
k1
$2\mathbf{k}_1 \neq \mathbf{k}_2 + \mathbf{k}_3'$
$|\mathbf{k}_3| = |\mathbf{k}_3'|$
f1
f3
f2=2f1-f3
f1
f3



Experimental condition:

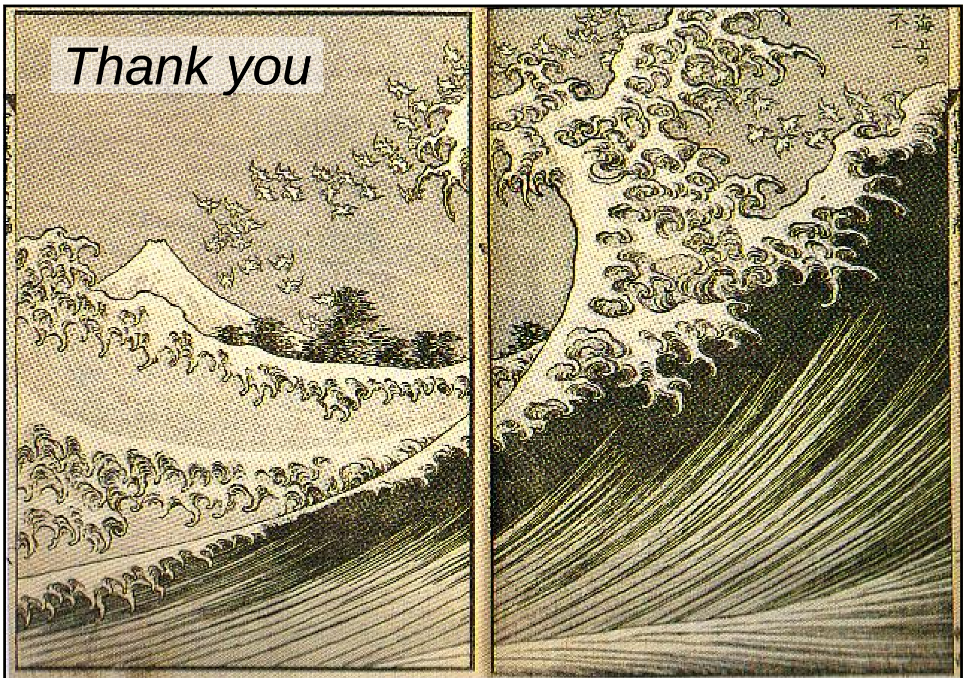
Tank Width b= 10 m
Wave length λ_2 = 6.67 m

b/λ_2=1.5

Jones 1984
Kit, Shemer & Miloh 1987
Yao, Tulin & Kolaini 1994

Sloshing Wave
Cross-wave
→ Tank resonance

⇒ 4 wave interaction has excited the sloshing wave



39「富嶽百景」二編　海上の不二　千葉市美術館蔵