EXTREME EVENTS IN CROSSING SEA STATES

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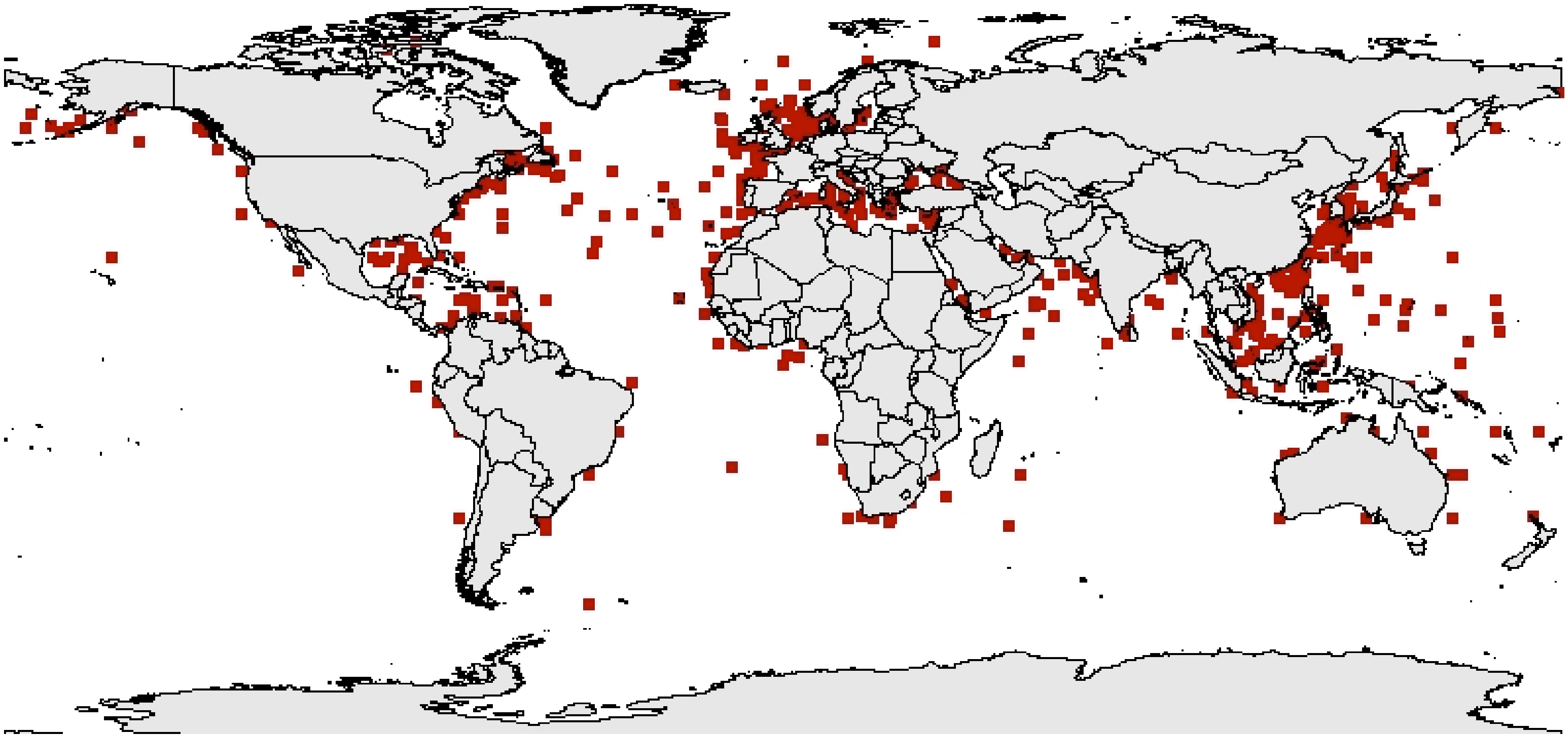
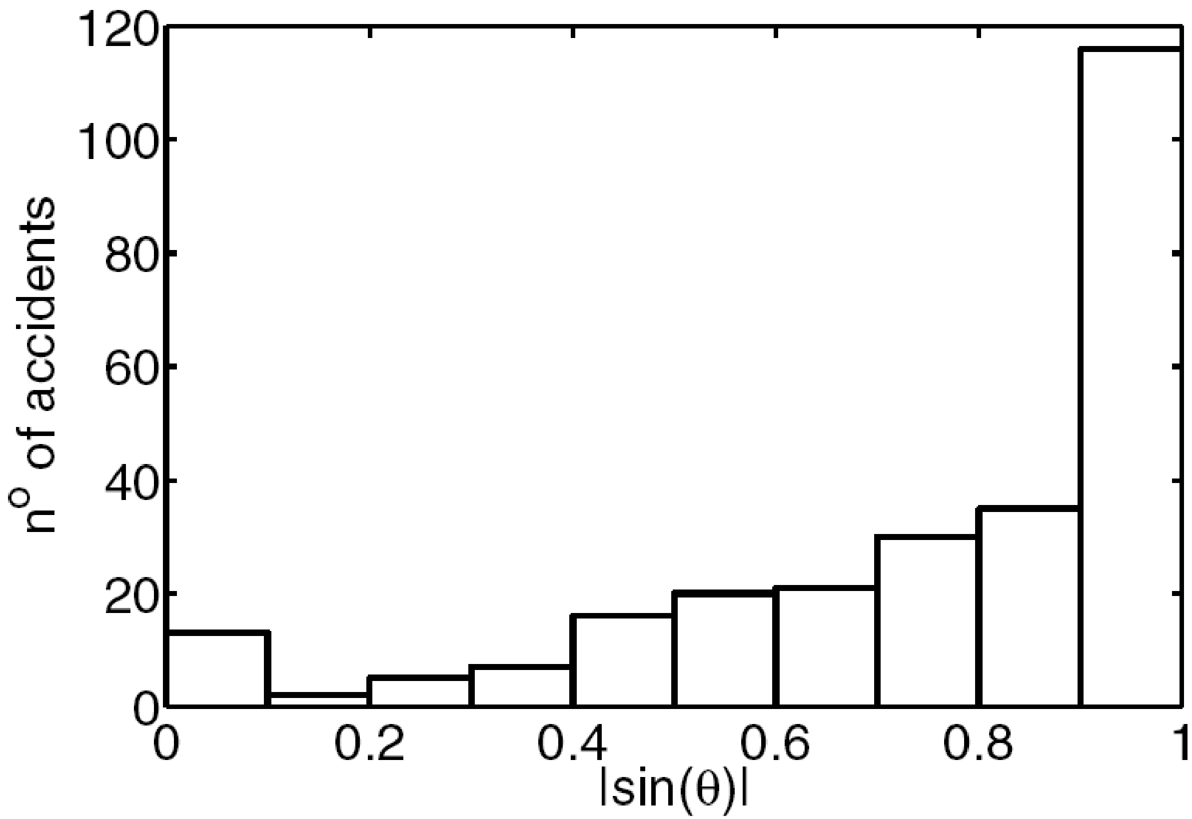
FIVE YEARS (FROM 1995-1999) OF SHIPS ACCIDENTS DUE TO HEAVY SEAS (total number = 650) (collected from Lloyd's Marine Information Service)



From Toffoli et *al*. "Towards the identification of warning criteria: analysis of a ship accident database", submitted to Applied Ocean Research 2005



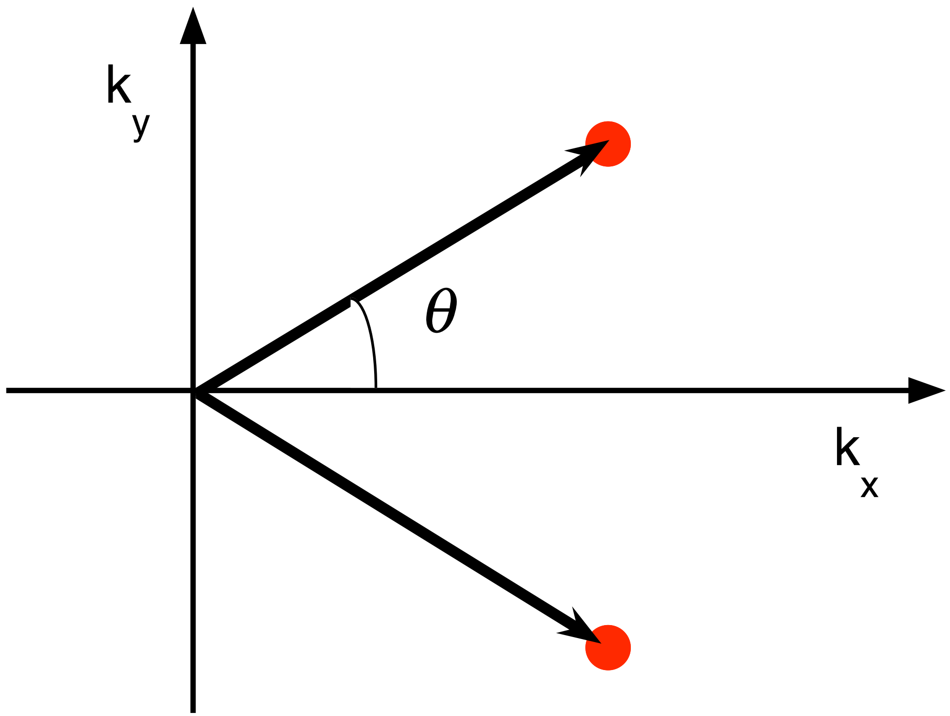
Time series from Draupner

From Lehner et al., Extreme wave observation from radar data sets, Waves 2005, Madrid

“The SAR images allow to analyze the spatial variations in respect to individual wave parameters as well as expectation values of the sea state to a spatial resolution of 30 m. Two crossing wave systems created by a meteorological front moving fast across an already existing wave system can be observed…”

OBJECTIVES OF THE PRESENT WORK:

Investigate theoretically and numerically the nonlinear dynamics and formation of extreme waves in crossing sea states



RESULTS

IN DEEP WATER

Bound modes -> Second order theory

Free modes -> CNLS equations

IN SHALLOW WATER

Numerical simulations of KP equation

SECOND ORDER THEORY (Longuet-Higgins, 1963)

$$\eta(\boldsymbol{x},t) = \sum_i a_i \cos(\psi_i) + \frac{1}{4}\sum_{i,j} a_i a_j [K_{ij}^{+} \cos(\psi_i + \psi_j) + K_{ij}^{-} \cos(\psi_i - \psi_j)]$$

where

$$\psi_i = \boldsymbol{k}_i \cdot \boldsymbol{x} - \omega_i t + \varphi_i$$

Hypotheses:

i) **Energy distributed around two peaks: k_1 and k_2**
ii) **Each energy distribution is narrow banded**
iii) **Infinite water depth**

$$\eta(\boldsymbol{x},t) = A_1\cos(\psi_1) + A_2\cos(\psi_2) + \\ + \frac{1}{4}\left[K_{11}^{+}A_1^2\cos(2\psi_1) + K_{22}^{+}A_2^2\cos(2\psi_2) + 2K_{12}^{+}A_1A_2\cos(\psi_1+\psi_2)\right]$$

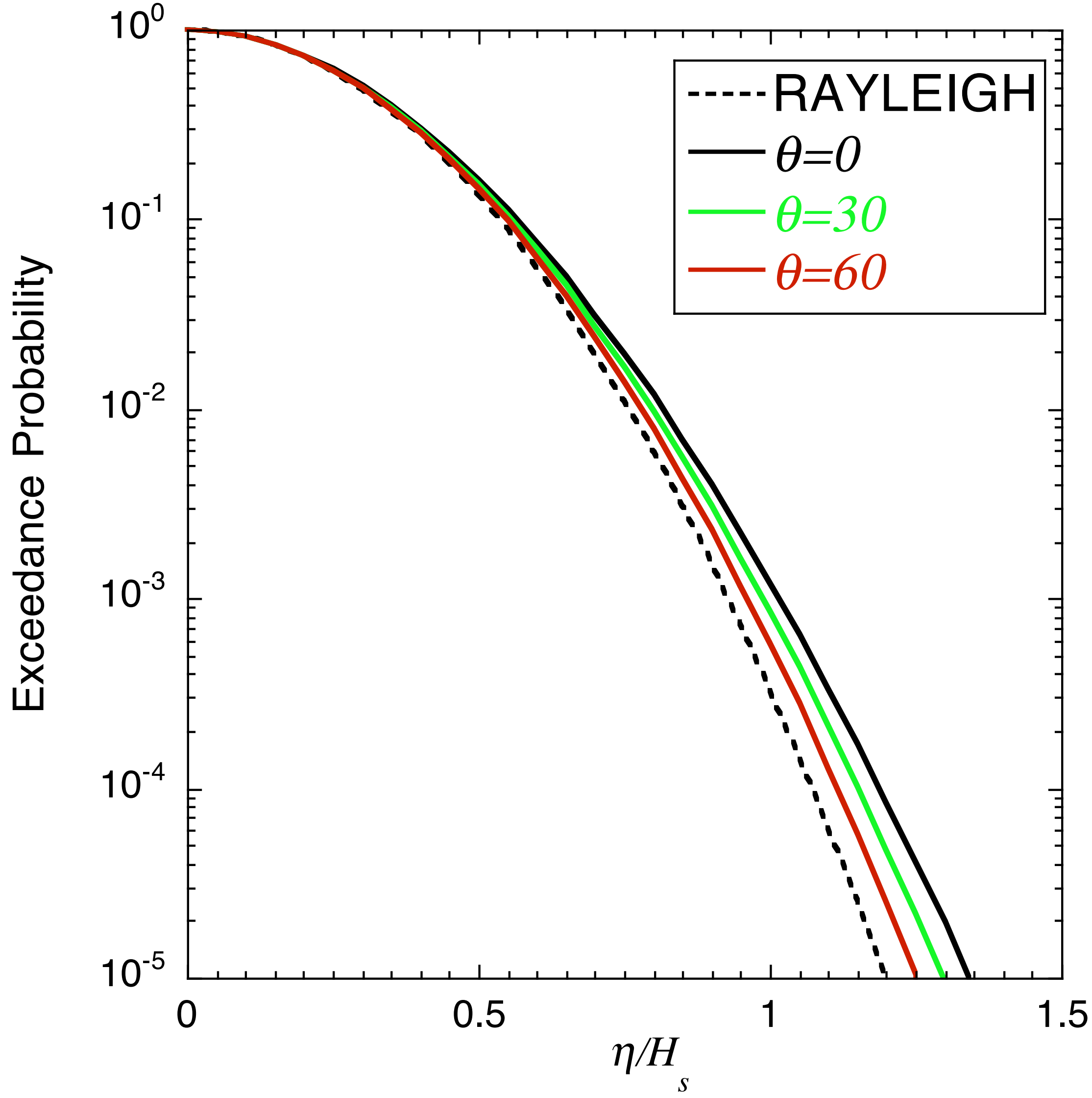
A_1 and A_2 are slowly varying wave envelope of the two wave trains in space and time

Following Tayfun, JGR 1980 and Socquet-Juglard *et al.*, JFM 2005, it is possible to find an upper bound for exceedence probability for wave crests:

$$P(\eta_c > x) = Exp\left[-\frac{2}{H_s^2 \alpha^2} \left(\sqrt{1 + 4x\alpha} - 1 \right)^2 \right]$$

with

$$\alpha = \frac{1}{16}\left(K_{11}^{+} + K_{22}^{+} + 2K_{12}^{+} \right)$$



RAYLEIGH
θ=0
θ=30
θ=60
Exceedance Probability
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
0.5
1
1.5
η/H_s

DYNAMICS OF FREE MODES

Evolution of free modes are described by the Zakharov equation

$$i\frac{\partial b_0}{\partial t} = \omega_0 b_0 + \int T_{1,2,3,4} b_1^* b_2 b_3 \delta_{0+1-2-3} dk_{123}$$

• consider the following decomposition:

$$b(\boldsymbol{k}) = A(\boldsymbol{k}-\boldsymbol{k_1})e^{-i\omega(k_1)t} + B(\boldsymbol{k}-\boldsymbol{k_2})e^{-i\omega(k_2)t}$$

with

$$\boldsymbol{k_1} = (k,l), \quad \boldsymbol{k_2} = (k,-l)$$

• suppose that both spectral distribution are narrow banded

COUPLED NLS EQUATIONS

$$\frac{\partial A}{\partial t}+C_x\frac{\partial A}{\partial x}+C_y\frac{\partial A}{\partial y}-i\left[\alpha\frac{\partial^2 A}{\partial x^2}+\beta\frac{\partial^2 A}{\partial y^2}-\gamma\frac{\partial^2 A}{\partial x\partial y}\right]+i\left[\xi|A|^2+2\zeta|B|^2\right]A=0$$

$$\frac{\partial B}{\partial t}+C_x\frac{\partial B}{\partial x}-C_y\frac{\partial B}{\partial y}-i\left[\alpha\frac{\partial^2 B}{\partial x^2}+\beta\frac{\partial^2 B}{\partial y^2}-\gamma\frac{\partial^2 B}{\partial x\partial y}\right]+i\left[\xi|B|^2+2\zeta|A|^2\right]B=0$$

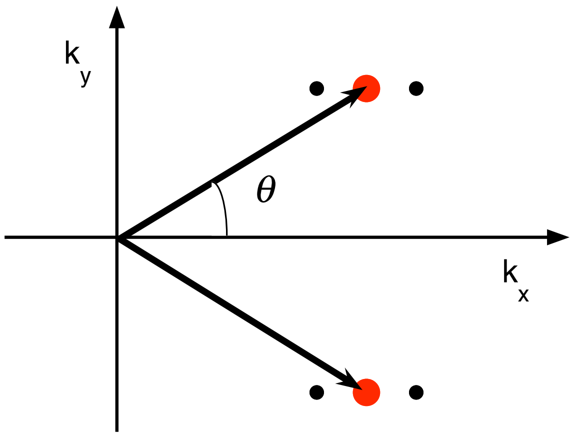
Coefficients are a function of *k* and *l*

Same equations have been derived in Hammack, J.L., Henderson, D.M. & Segur, H., JFM 2005

LINEAR STABILITY ANALYSIS

(similar work but with different methodology has been performed by Badulin et al, JFM 1995, Ioualalen and Kharif, JFM 1994)

- consider perturbations only a function of k_x

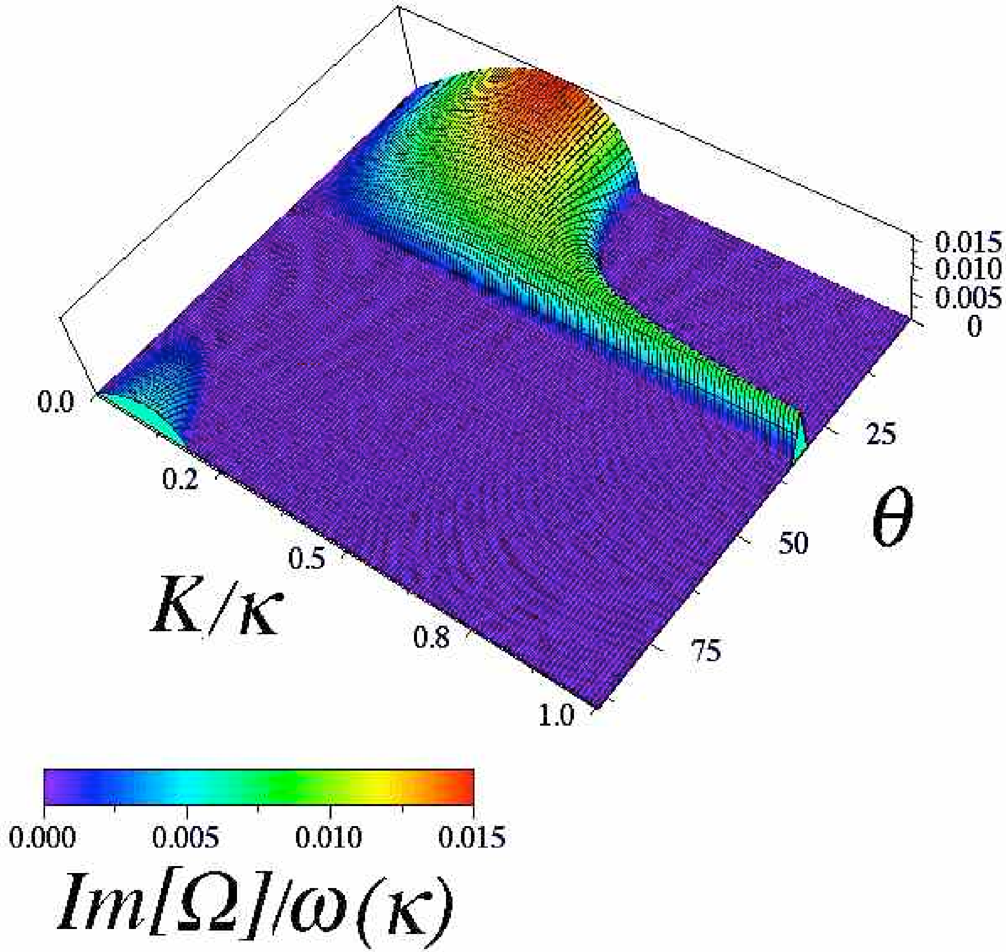


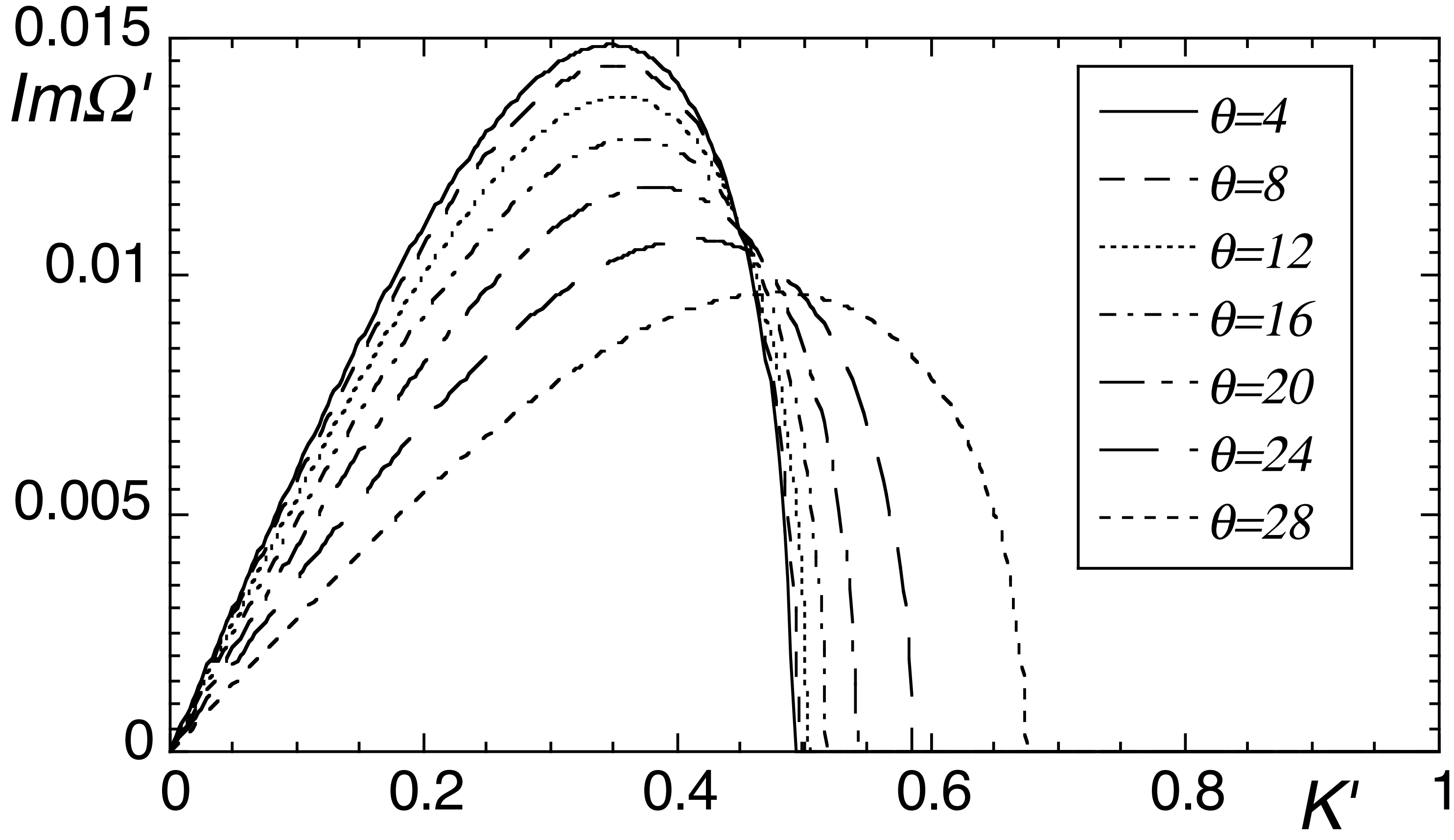
$$\frac{\partial A}{\partial t} + -i\alpha \frac{\partial^2 A}{\partial x^2} + i\left[\xi |A|^2 + 2\zeta |B|^2\right] A = 0$$

$$\frac{\partial B}{\partial t} - i\alpha \frac{\partial^2 B}{\partial x^2} + i\left[\xi |B|^2 + 2\zeta |A|^2\right] B = 0$$

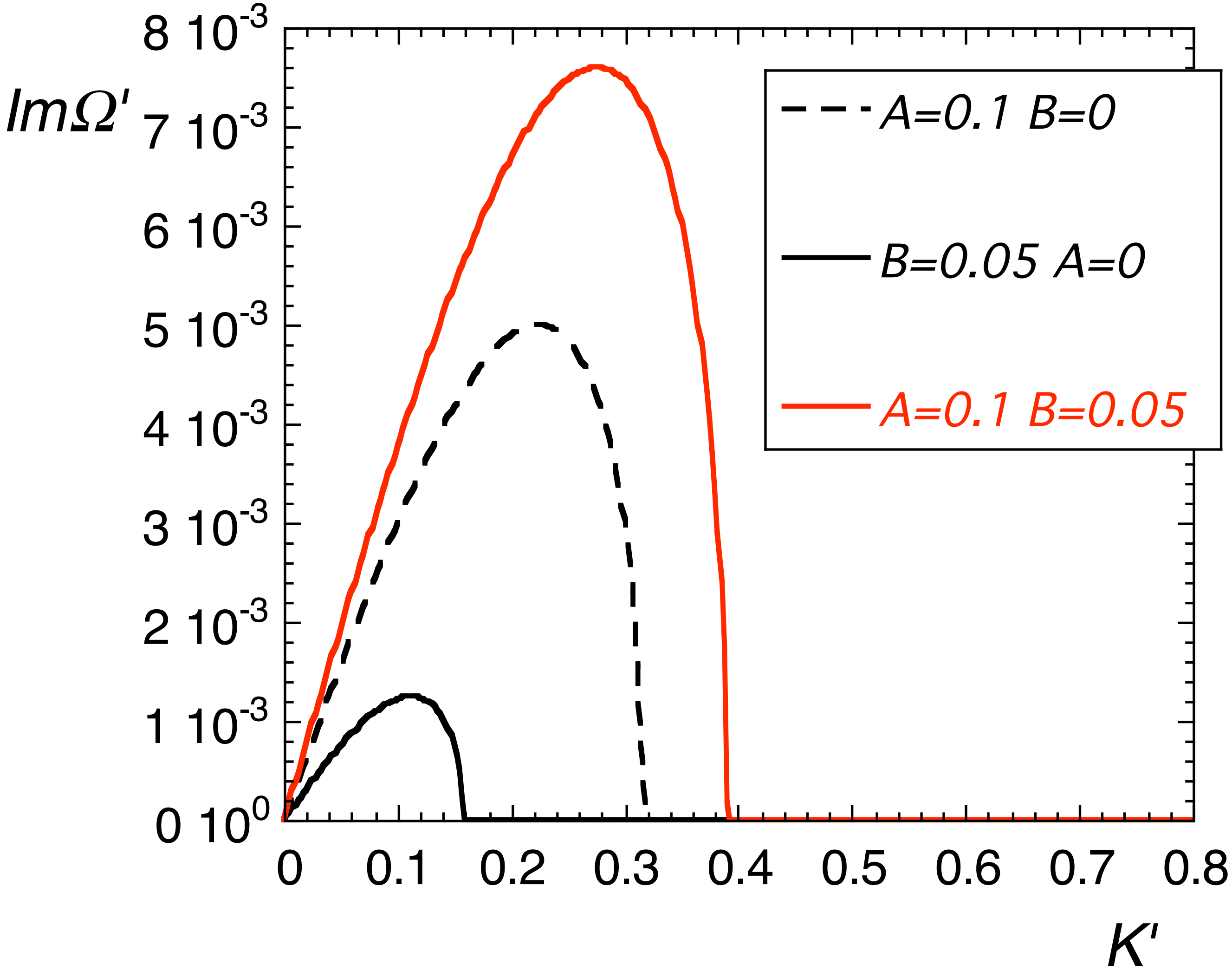
DISPERSION RELATION

$$\Omega = \pm\sqrt{\alpha K^2\left[\xi\left(A_0^2+B_0^2\right)+\alpha K^2\right]\pm\sqrt{\xi^2\left(A_0^2-B_0^2\right)^2+16\zeta^2A_0^2B_0^2}}$$





0.015
ImΩ'
0.01
0.005
0
0
0.2
0.4
0.6
0.8
1
K'
θ=4
θ=8
θ=12
θ=16
θ=20
θ=24
θ=28



$Im\Omega'$
8 10^{-3}
7 10^{-3}
6 10^{-3}
5 10^{-3}
4 10^{-3}
3 10^{-3}
2 10^{-3}
1 10^{-3}
0 10^{0}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
K'
A=0.1 B=0
B=0.05 A=0
A=0.1 B=0.05

NUMERICAL SIMULATIONS OF COUPLED NLS

$A=0.1\ m$

$B=0.05\ m$

$\kappa=1\ m^{-1}$

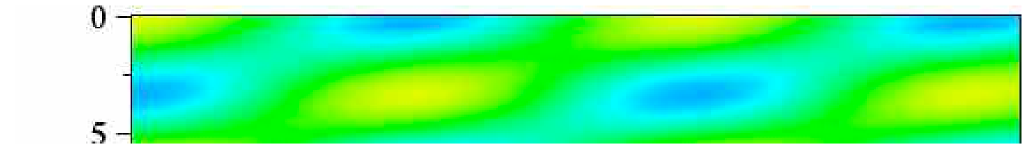
$\varepsilon_A=0.1$

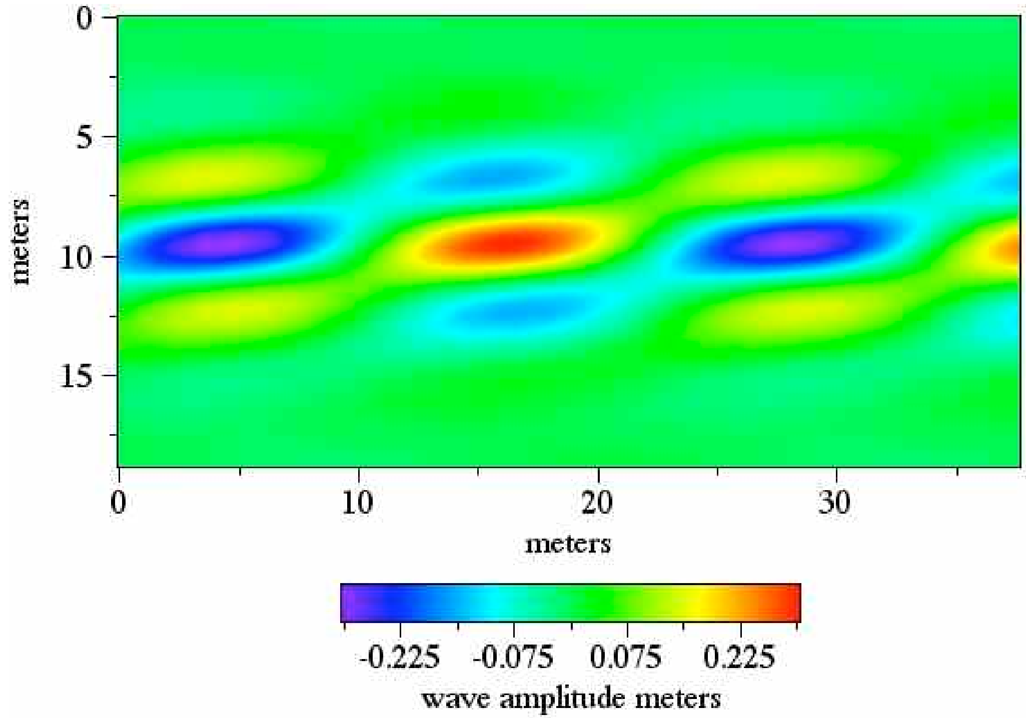
$\varepsilon_B=0.05$

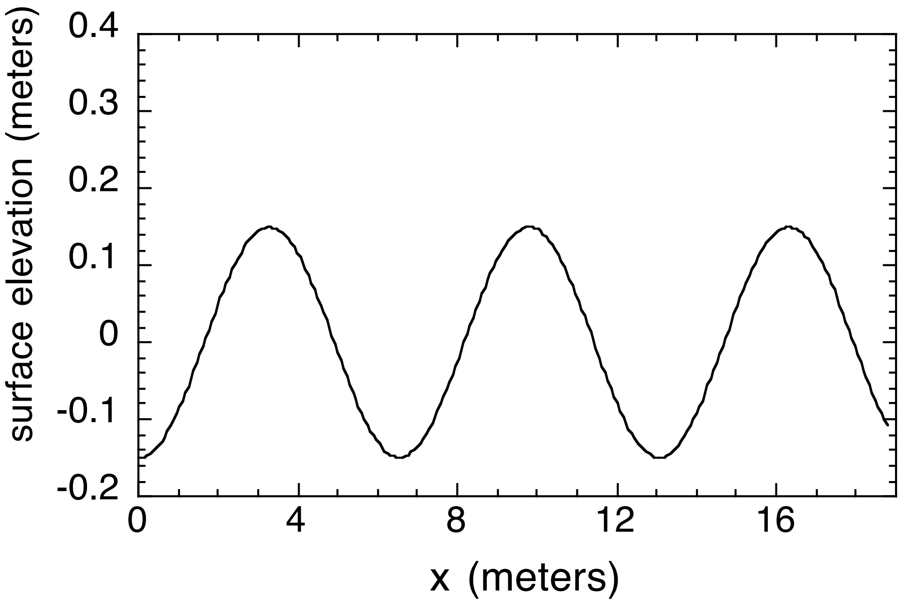
$K/\kappa=0.3333$ –> 3 *waves under the perturbation*

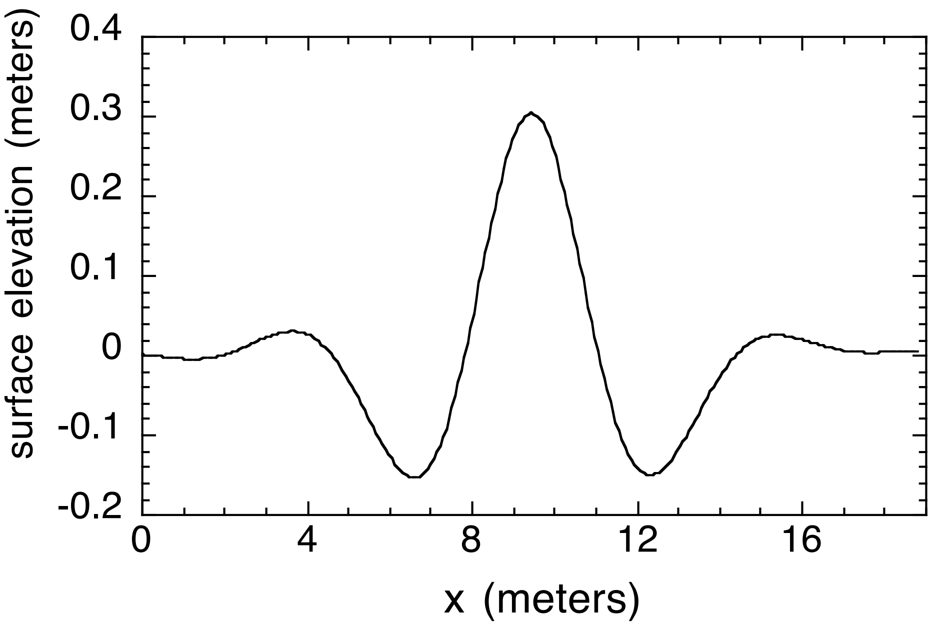
$\theta=15^o$

Note that each wave train, taken independently, is stable to the imposed side – band perturbation


0
5

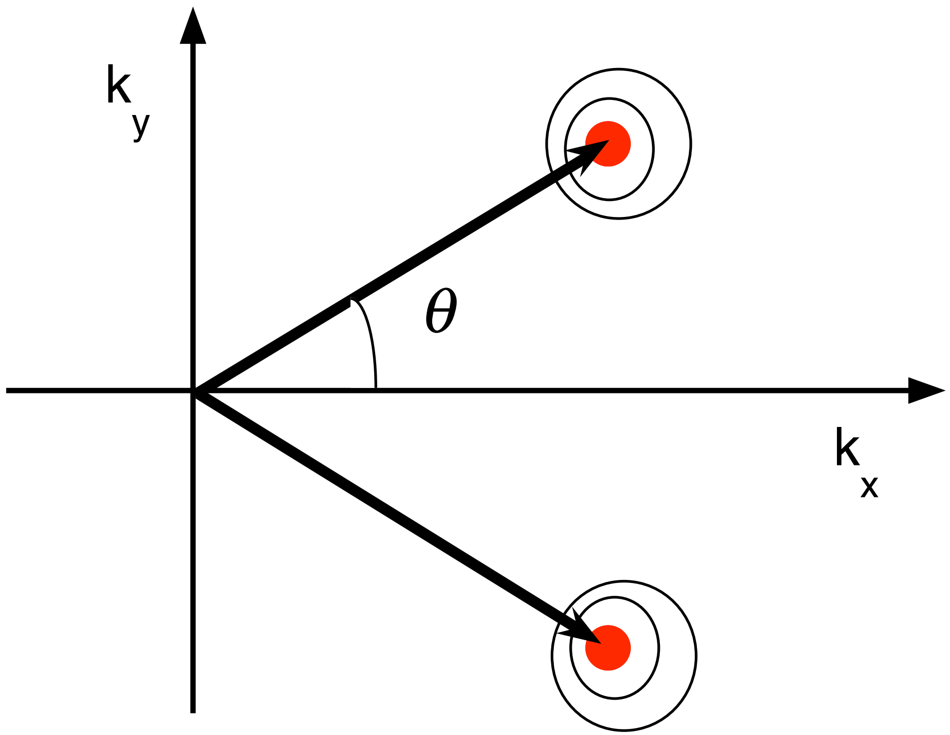

0
5
10
15
meters
0
10
20
30
meters
-0.225
-0.075
0.075
0.225
wave amplitude meters


surface elevation (meters)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
4
8
12
16
x (meters)


surface elevation (meters)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
4
8
12
16
x (meters)

SHALLOW WATER

NUMERICAL SIMULATIONS OF THE KP EQUATION WITH RANDOM WAVES



Two Jonswap spectra each with directional spreading

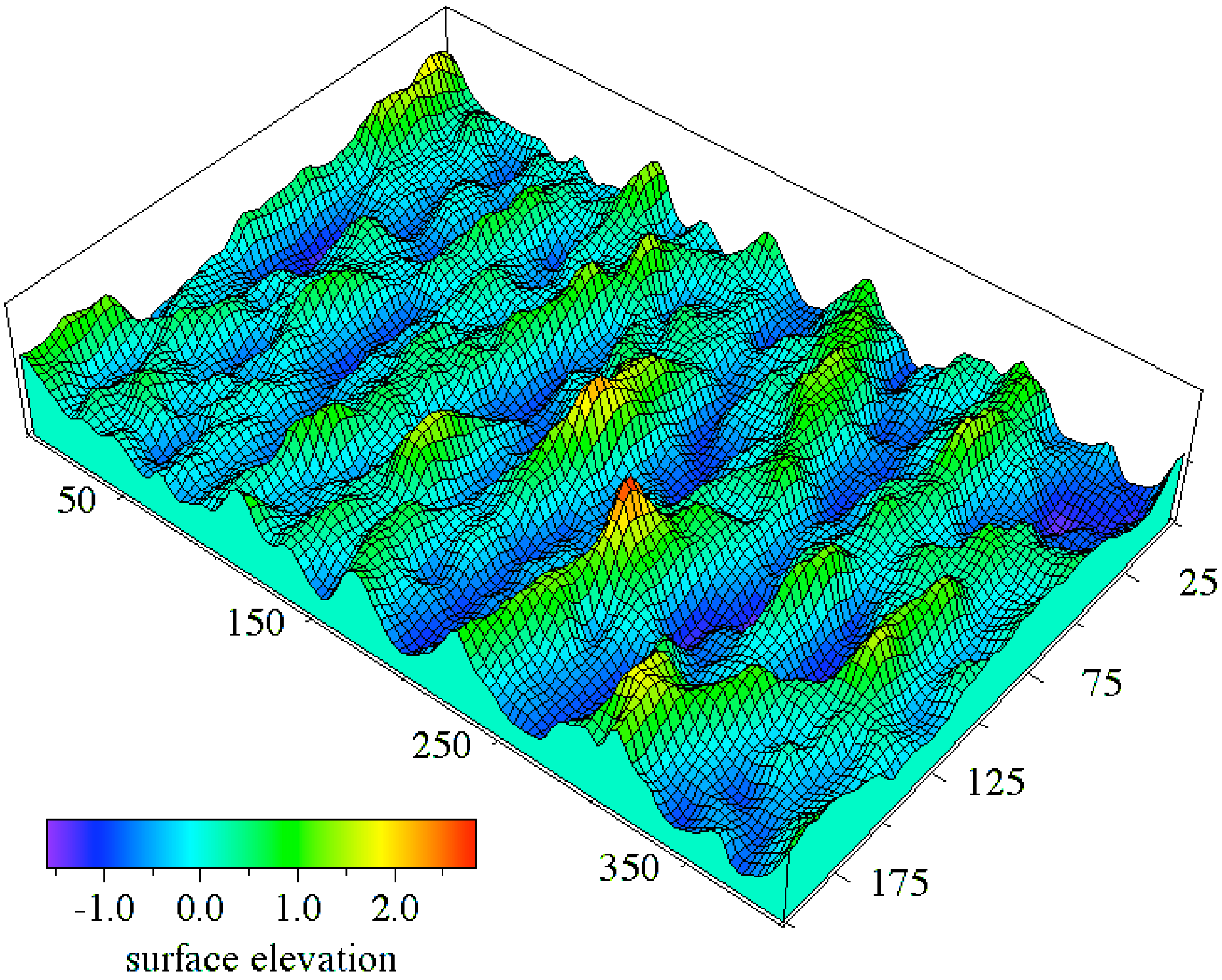
Tp=7 s

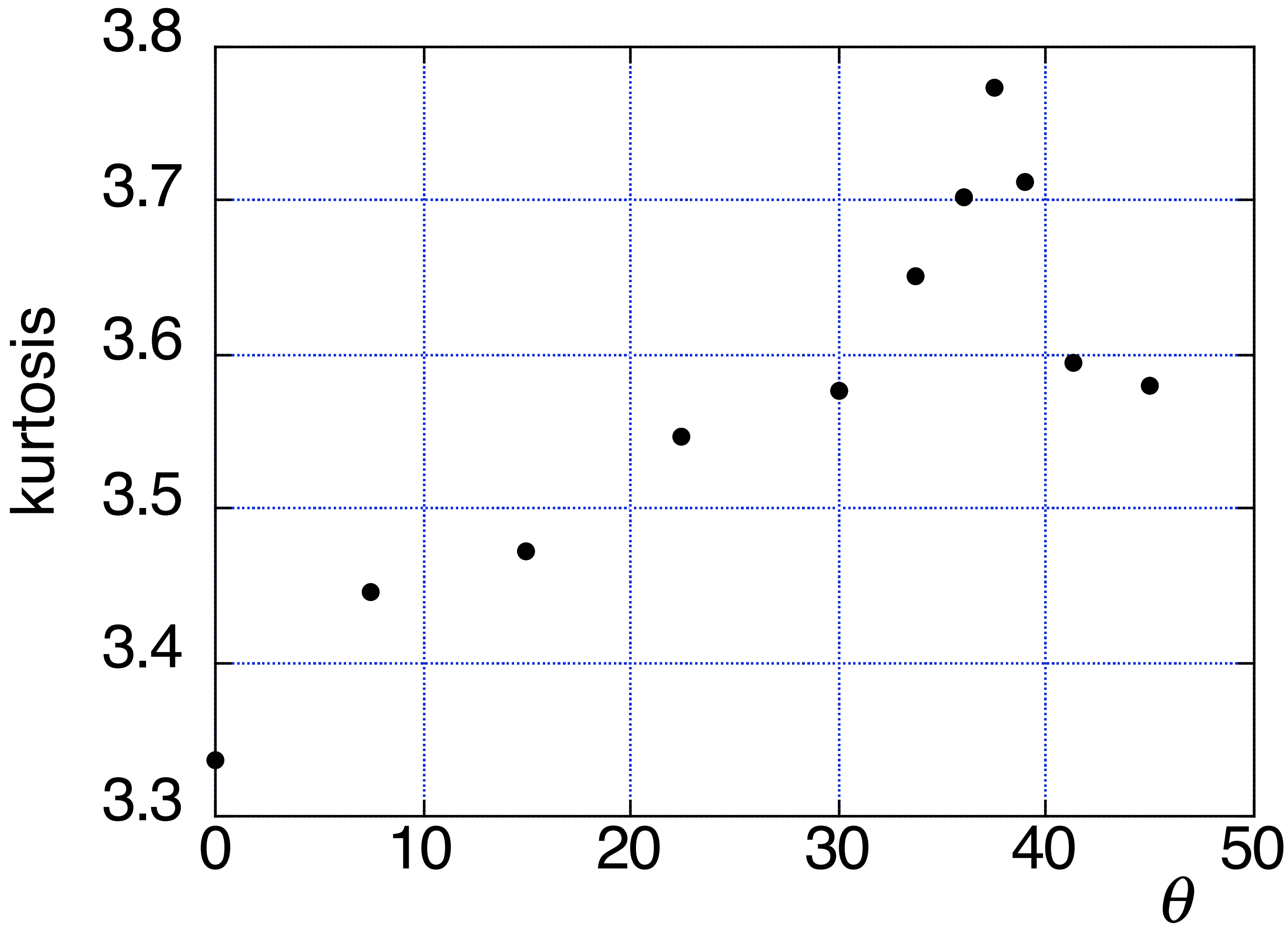
γ=*3.3*

domain: 400 m x *200 m*

$$Ur = \frac{H_s k_p}{\left(k_p d\right)^3} = 0.7$$

different simulations for different angles


50
150
250
350
25
75
125
175
-1.0
0.0
1.0
2.0
surface elevation


3.8
3.7
3.6
3.5
3.4
3.3
kurtosis
0
10
20
30
40
50
θ