Nonlinear modeling of the geometry and kinematics of extreme waves

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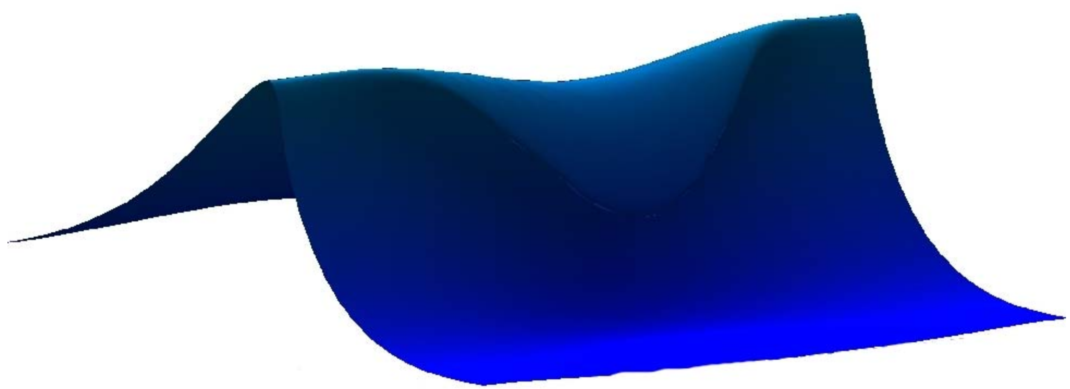


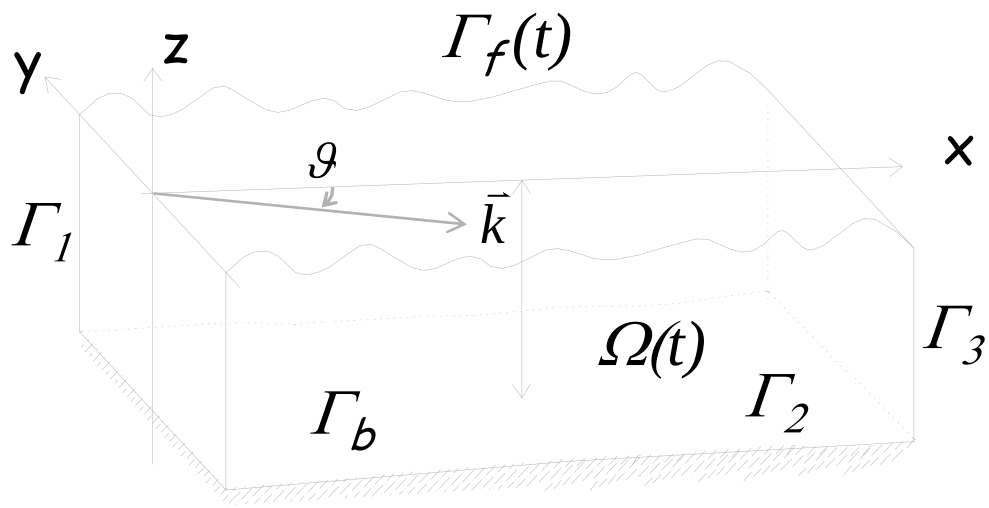
Laboratory experiments

University of Tokyo – Ocean Engineering Tank
Waseda et al.

Examples of numerical wave tanks

- Xue et al. (2001) – Boundary element method
- Grilli et al. (2001) – Boundary element method
- Fructus et al. (2005) – (Grue's talk yesterday)
- Bateman et al. (2001) – Fourier expansions
- Fuhrman et al. (2004) – « Boussinesq » model
- Bonnefoy et al. (2004) – Spectral method
- …
- Dyachenko et al. (2002) – Conformal transformation (2D and semi 3D)

Governing Equations and Boundary Conditions



Potential flow : $\nabla^2 \phi = 0$ in Ω

Boundary conditions :

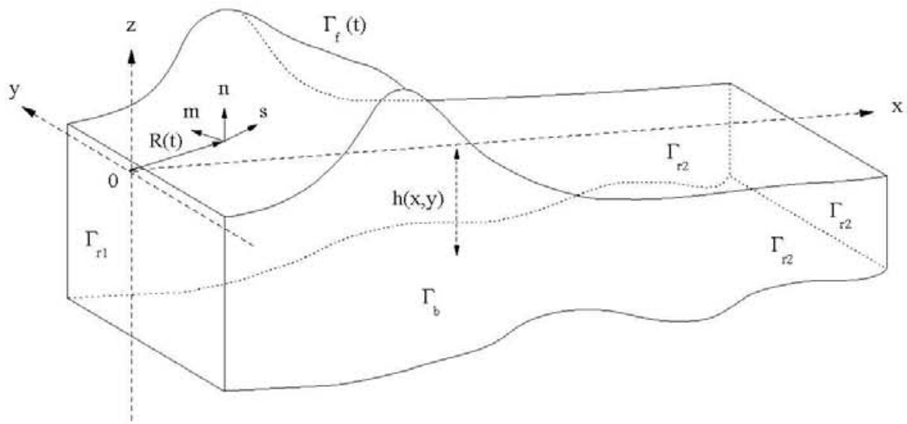
- *free surface* : kinematic and dynamic boundary conditions

  $DR / Dt = \mathbf{u}$; $D\phi / Dt = -gz + 1/2\ \nabla\phi . \nabla\phi - p_f/\rho$ on Γ_f

- *fixed boundaries* : no-flow boundary condition $\partial\phi / \partial n = 0$ on Γ_b

- *moving boundaries* : specified motion (for example, wave-maker motion on the open sea boundary)

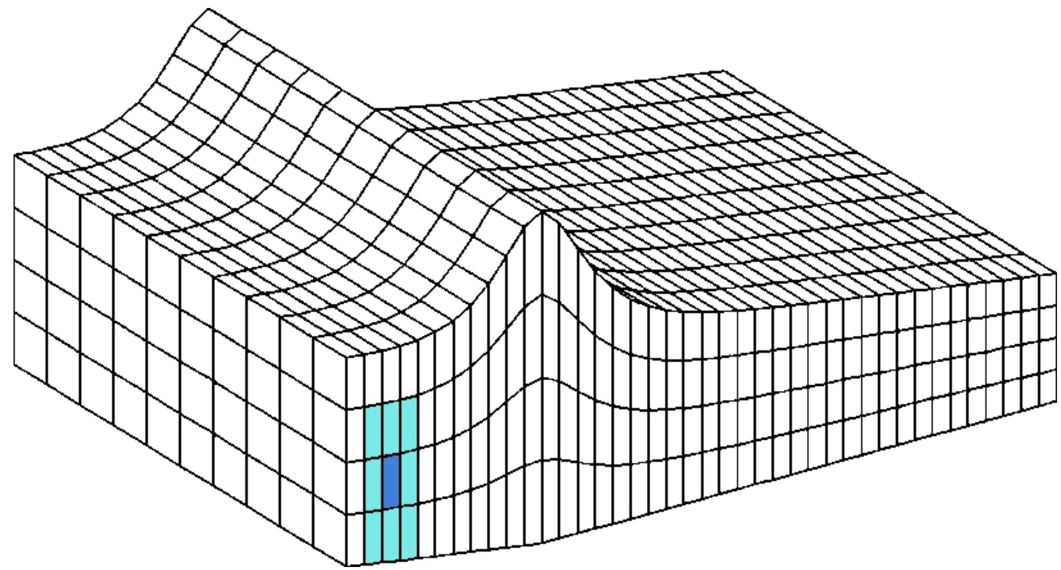
Domain of computation and governing equations



Green's second identity transforms Laplace's equation into a Boundary Integral Equation (BIE),

$$\alpha(\boldsymbol{x}_l)\,\phi(\boldsymbol{x}_l) = \int_{\Gamma(t)} \left\{ \frac{\partial \phi}{\partial n}(\boldsymbol{x})\, G(\boldsymbol{x}, \boldsymbol{x}_l) - \phi(\boldsymbol{x})\, \frac{\partial G}{\partial n}(\boldsymbol{x}, \boldsymbol{x}_l) \right\} \mathrm{d}\Gamma$$

Boundary element method

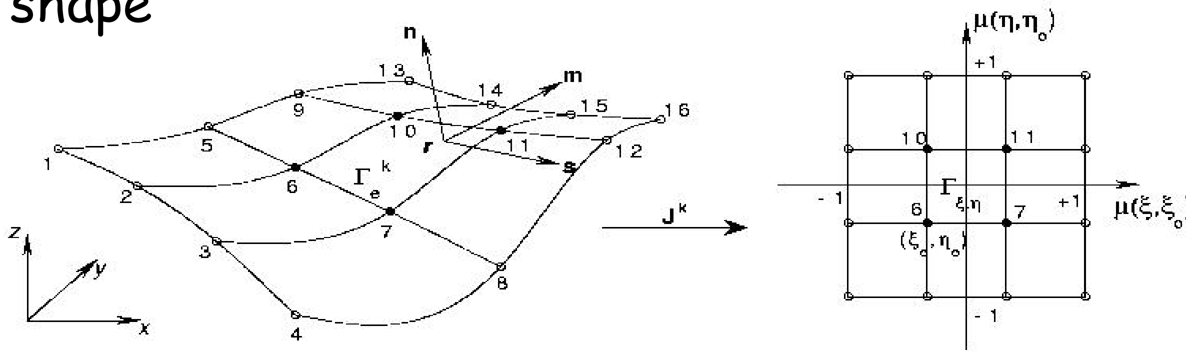


- Discretization of the boundary into N collocation nodes (typically several thousands)

- Use of M higher-order elements (typically several thousands)

- Local interpolation of the geometry and field variables using shape functions

m=16 C^2 continuity at the intersection between 2 elements

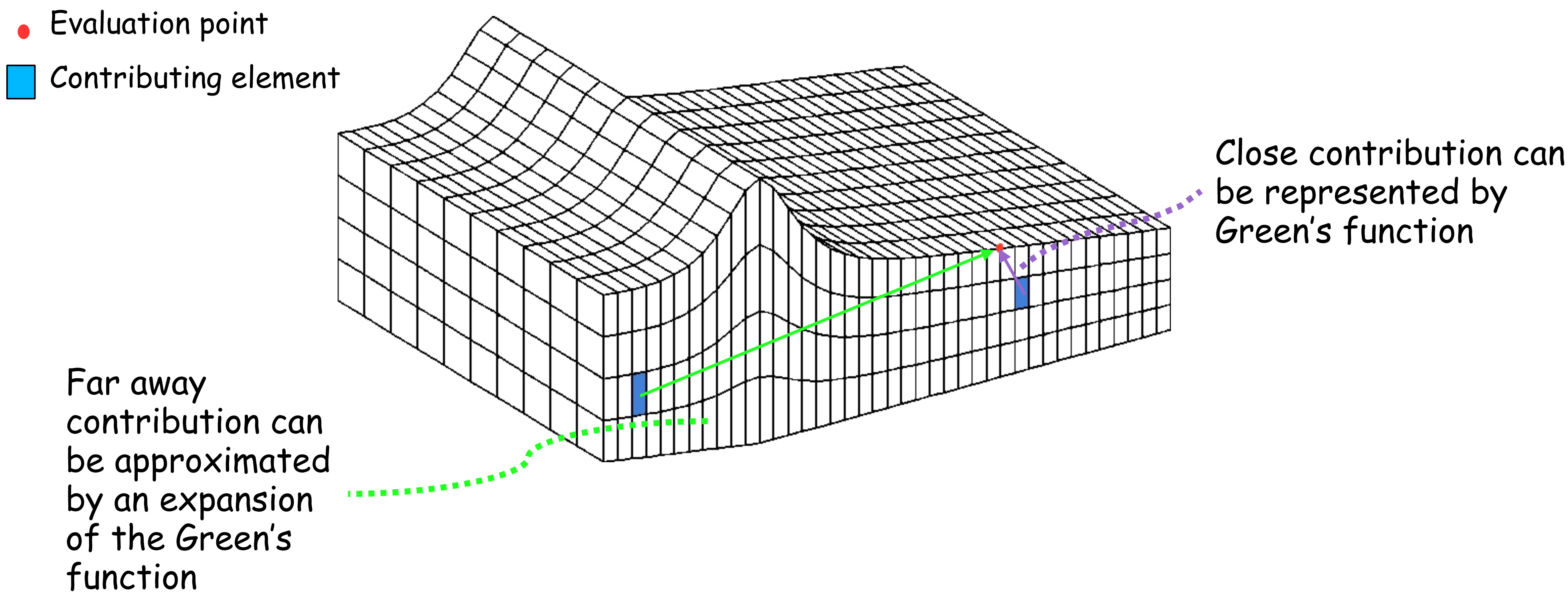


Time integration

Free surface time updating is achieved using a mixed Eulerian-Lagrangian (MEL) method, based on explicit 2nd-order Taylor series expansions for both the potential and the geometry : Lagrangian trajectories of free surface nodes

The time step Dt is adapted at each time as a function of the minimum distance between two nodes on the free surface and a constant Courant number. The time stepping scheme is explicit.

An additional approximation



Approximated Boundary Integral Equation

Greengard, Rokhlin (mid 1980s)

Fast Multipole Algorithm

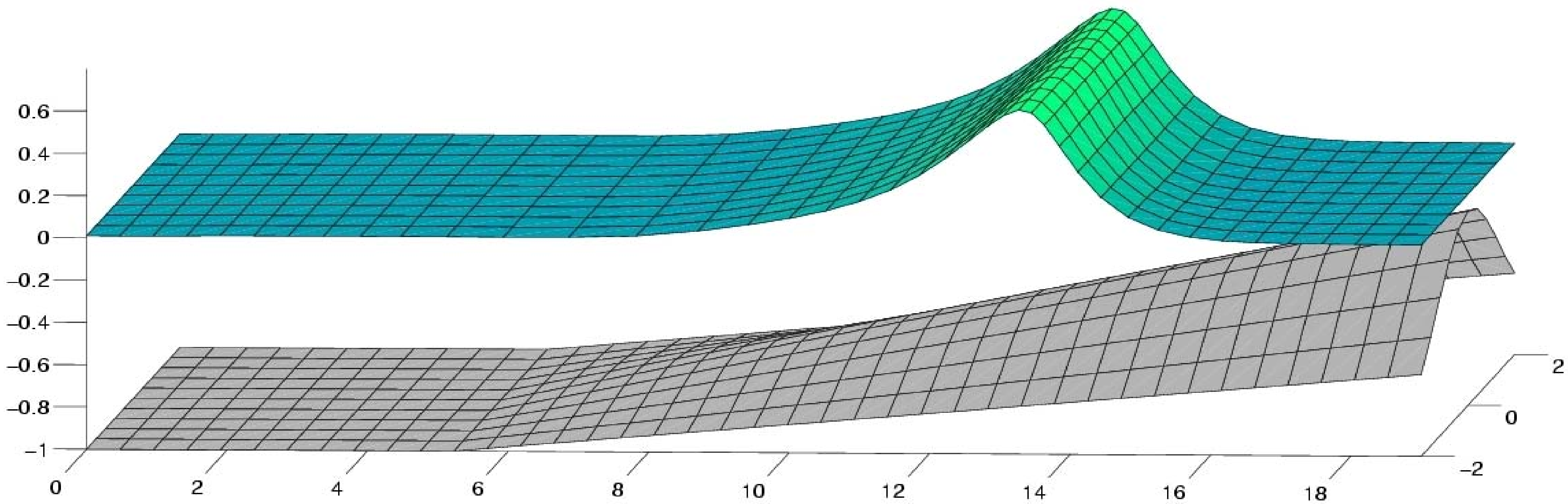
The FMA is based on the principle that the Green's function can be expanded in separated variables when the source point $\boldsymbol{x}_l$ and the evaluation point $\boldsymbol{x}$ are far enough from each other on the boundary. Thus, one can write for a point O (origin of the expansion) close to $\boldsymbol{x}$ and far from $\boldsymbol{x}_l$,

$$G(\boldsymbol{x}, \boldsymbol{x}_l) \approx \frac{1}{4\pi} \sum_{k=0}^{p} \sum_{m=-k}^{k} \rho^k Y_k^{-m}(\alpha, \beta) \frac{Y_k^m(\theta, \varphi)}{r^{k+1}},$$

where $O\boldsymbol{x} = (\rho, \alpha, \beta)$ and $O\boldsymbol{x}_l = (r, \theta, \varphi)$ in spherical coordinates. Functions $Y_k^{\pm m}$ are the spherical harmonics defined from the Legendre polynomials.

Comparisons

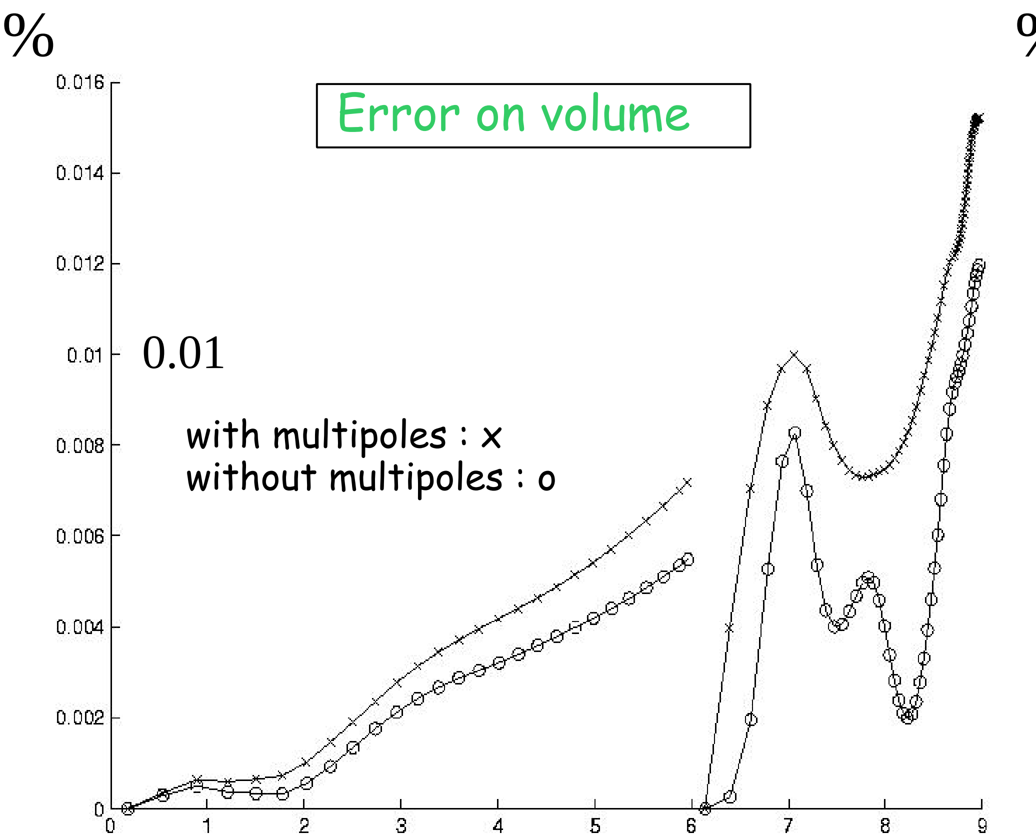
Do we obtain the same solution with the new method for the same level of approximation ?

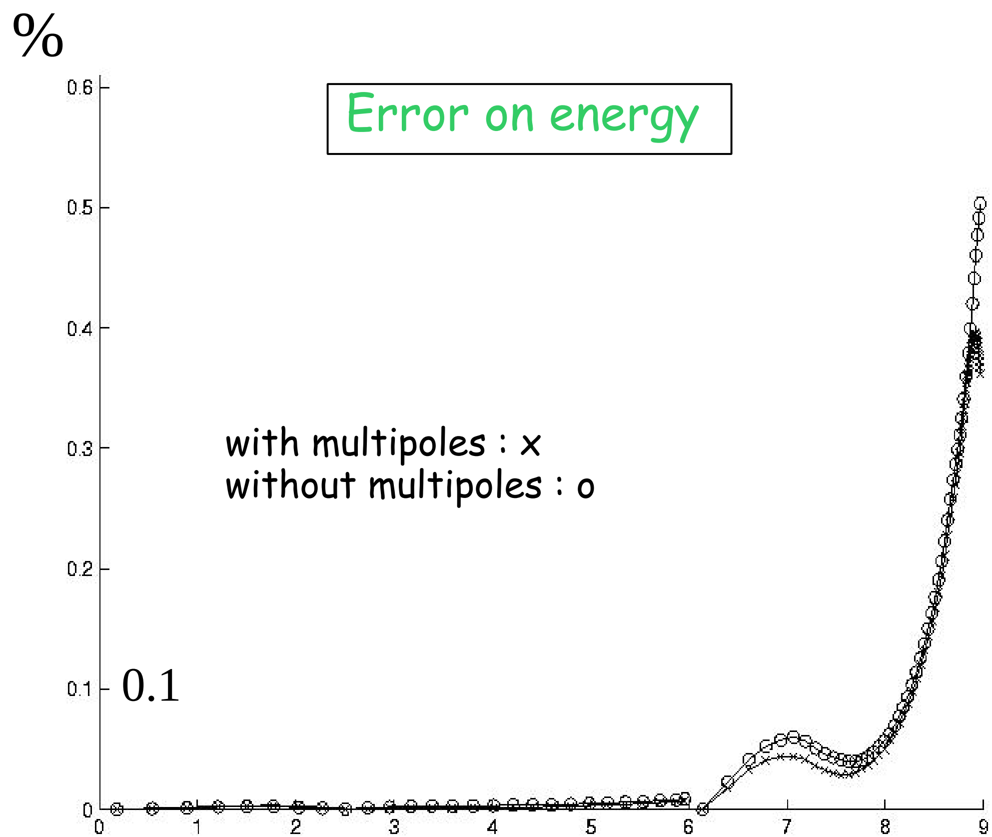


But with a smaller time ?

Comparisons

Conservation of volume and energy (mesh 40*10*4)
(breaking of solitary wave)





Parameters : 5 levels, multipoles degree = 8

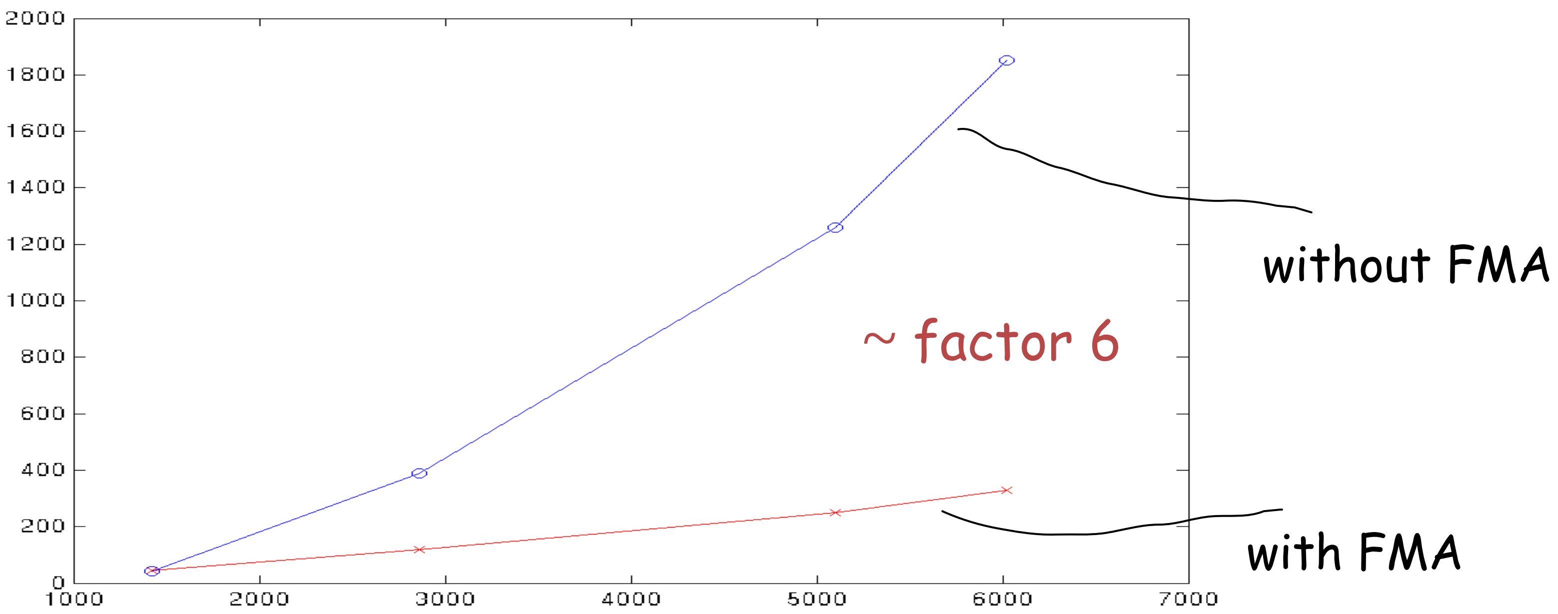
Same wave profiles

Comparisons

Computing time
solution of the first linear system
(solitary wave case)

2000

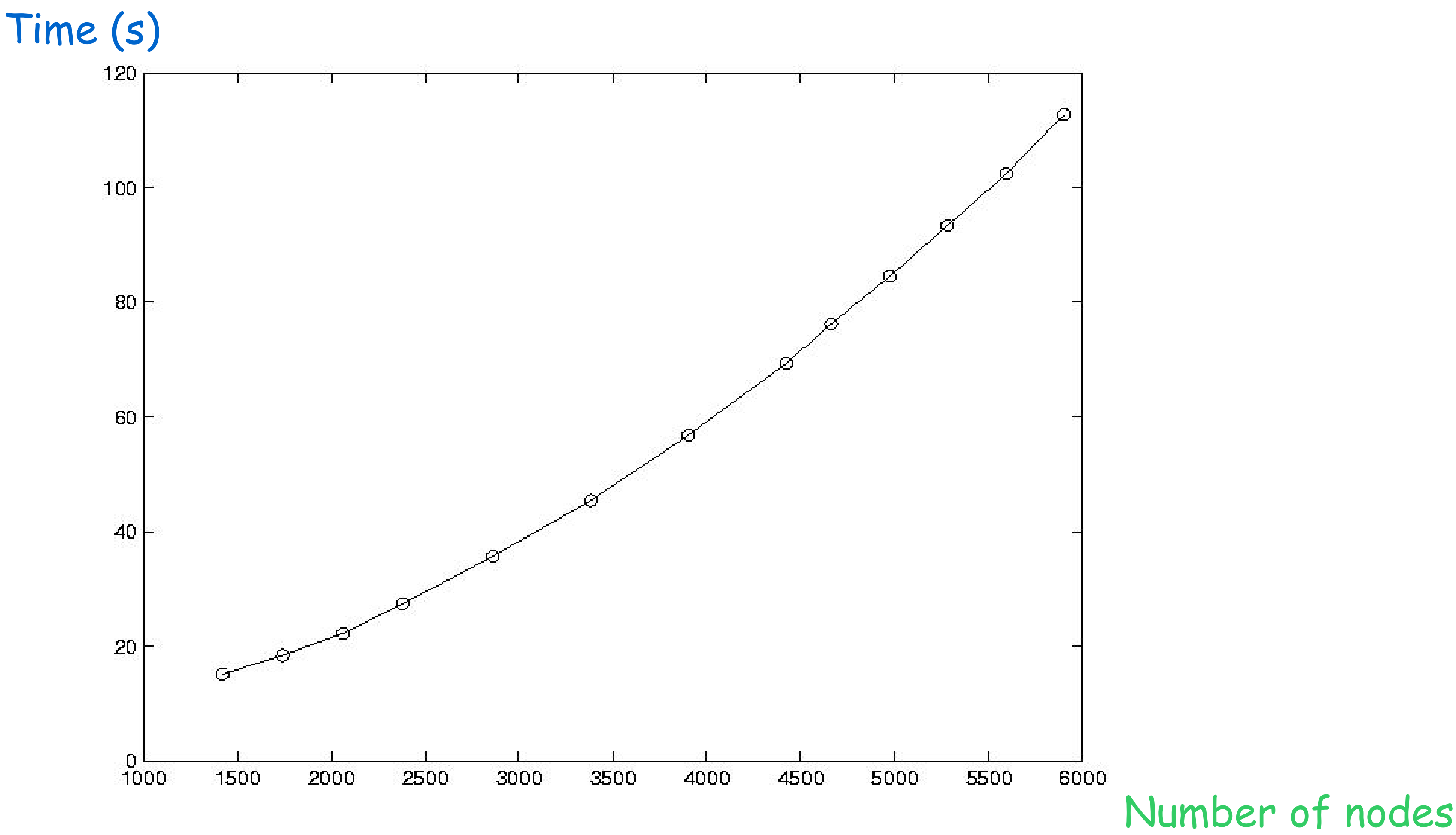
Time (s)



3000

Number of nodes

Comparisons



Complexity ~ linear above 4500 nodes

Description of the wavemaker

The NWT is a rectangular basin with a flat bottom at depth d. Laterally, the NWT is bounded by fixed or moving boundaries. At one extremity, a snake wavemaker is implemented.

The wavemaker stroke S_o is specified as the linear superposition of N_θ sinusoidal components of amplitude a_n and direction θ_n:

$$S_o(y,t) \quad = \quad \sum_{n=1}^{N_\theta} a_n \cos\{k_n(y \sin\theta_n - x_f \cos\theta_n) - \Omega_n t\}$$

where k_n and ω_n denote the wavenumber and angular frequency of each component, respectively. They satisfy the linear dispersion relation

$$\Omega_n^2 \quad = \quad g\, k_n \tanh(k_n d).$$

x_f is the focusing distance for the waves in front of the wavemaker. Angles θ_n are uniformly distributed in the range $[-\theta_{\max}, \theta_{\max}]$.

Description of the snake wavemaker

Generation of converging wave components whose amplitudes are given through the transfer function for such a flap wavemaker:

$$A_n = a_n \frac{4\sinh^2(k_n d)}{(2k_n d + \sinh(2k_n d))\cos\theta_n}.$$

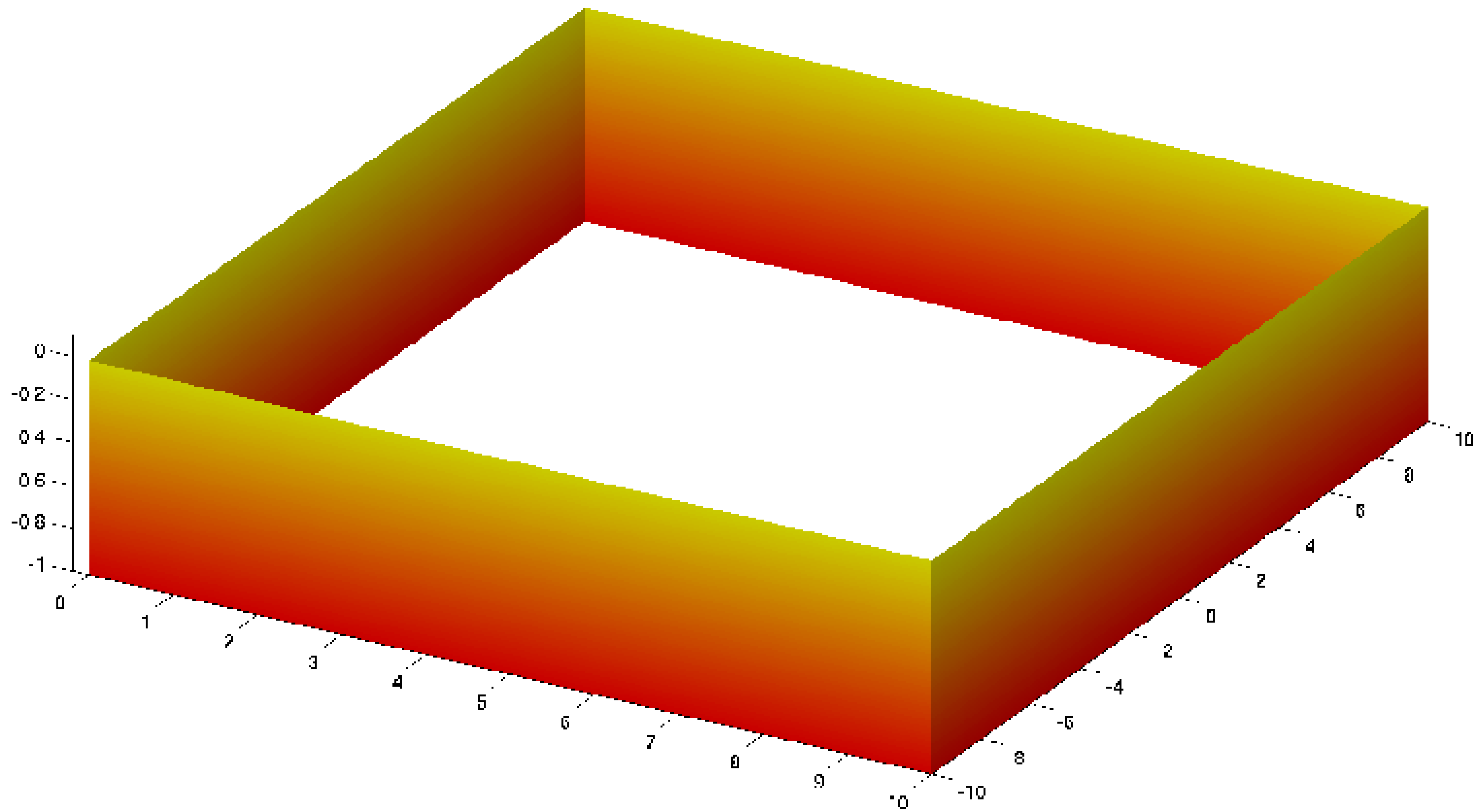
At the linearly assumed focus point, the total amplitude is denoted by

$$A = \sum A_n$$

Values of wavemaker parameters

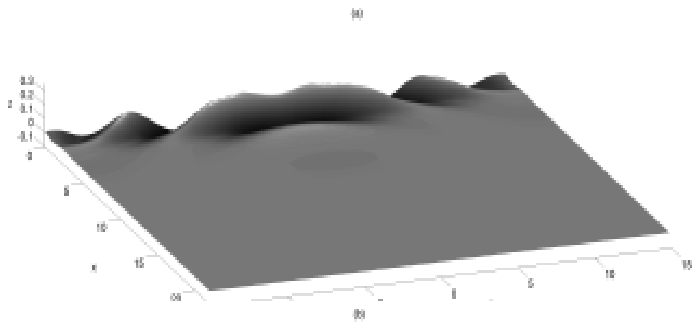
* Superposition of 30 components having identical properties, but with directions varying between -45 and 45 degrees.
* Unit length = water depth d, unit time = $\sqrt{d/g}$
* Each component has a frequency $\Omega = 0.9$ rad/s, which gives a wavelength $L = 72$ m and a linear velocity $c = 10.28$ m/s for a water depth of $d = 20$ m.
* The amplitude of each individual component is fixed to $a = 0.2$ m, yielding a steepness of $ka = 0.017$.
* The energy focusing point is at the distance $x_f = 250$ m from the wavemaker. The amplitude at that point is $A = 10.5$ m.
* Tank dimensions: length=440 m, width=600 m. Number of elements: 90 x 70 x 4. In order to obtain the overturning phase, the resolution is improved to 120 elements.

Motion of snake wavemaker

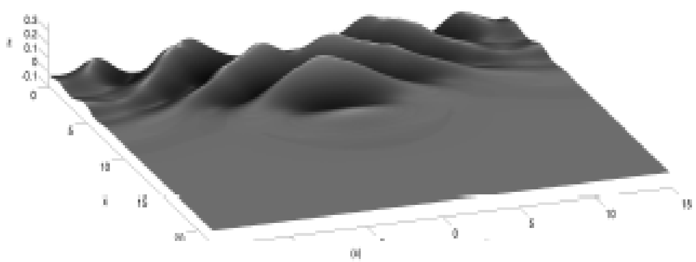


The wave-maker motion follows Dalrymple's linear theory so that waves focus at a given point inside the wave tank.

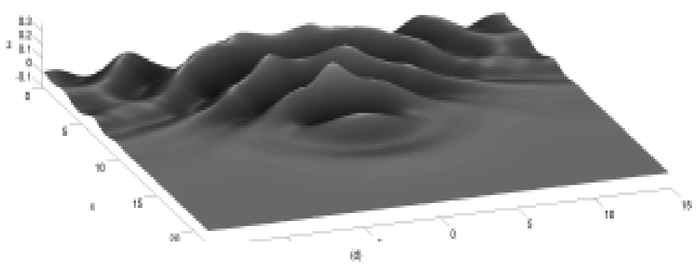
Free-surface evolution



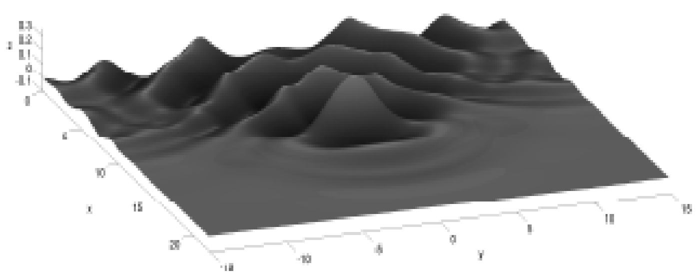
T = 10.66



T = 23.23

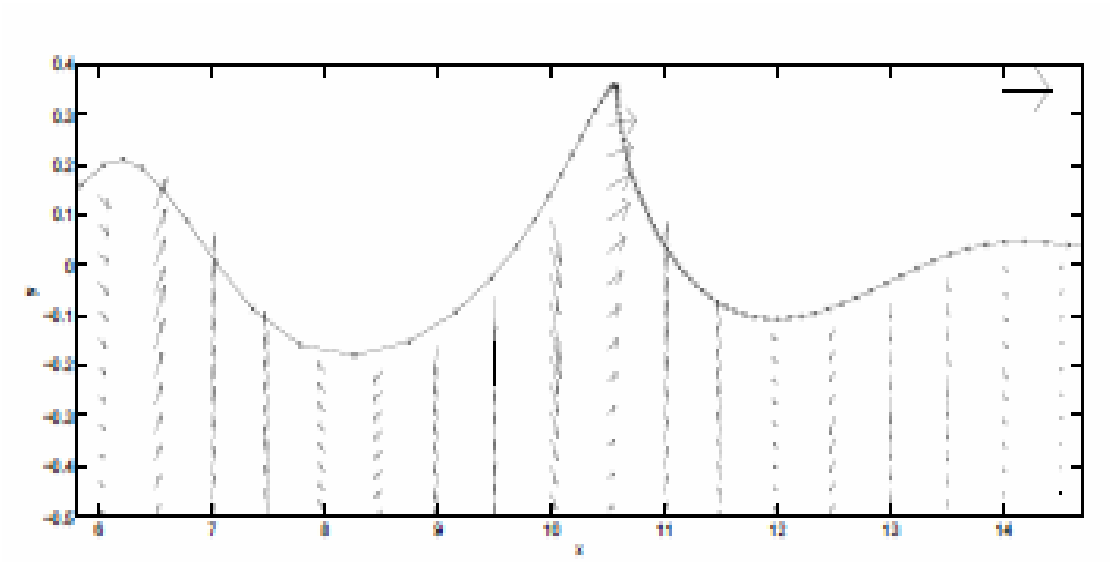


T = 30.37



T = 33.76

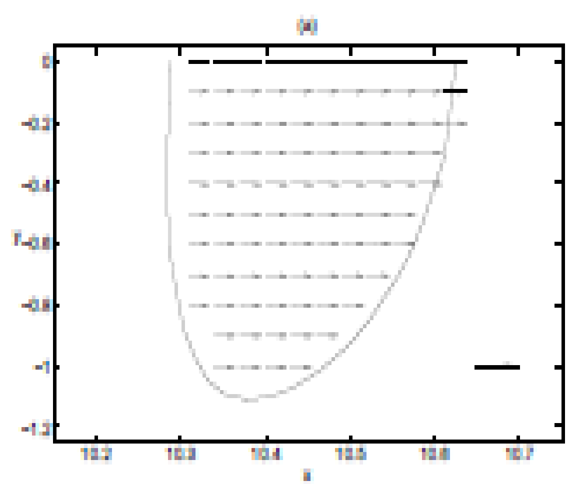
Vertical and horizontal slices



T = 33.76

Y = 0

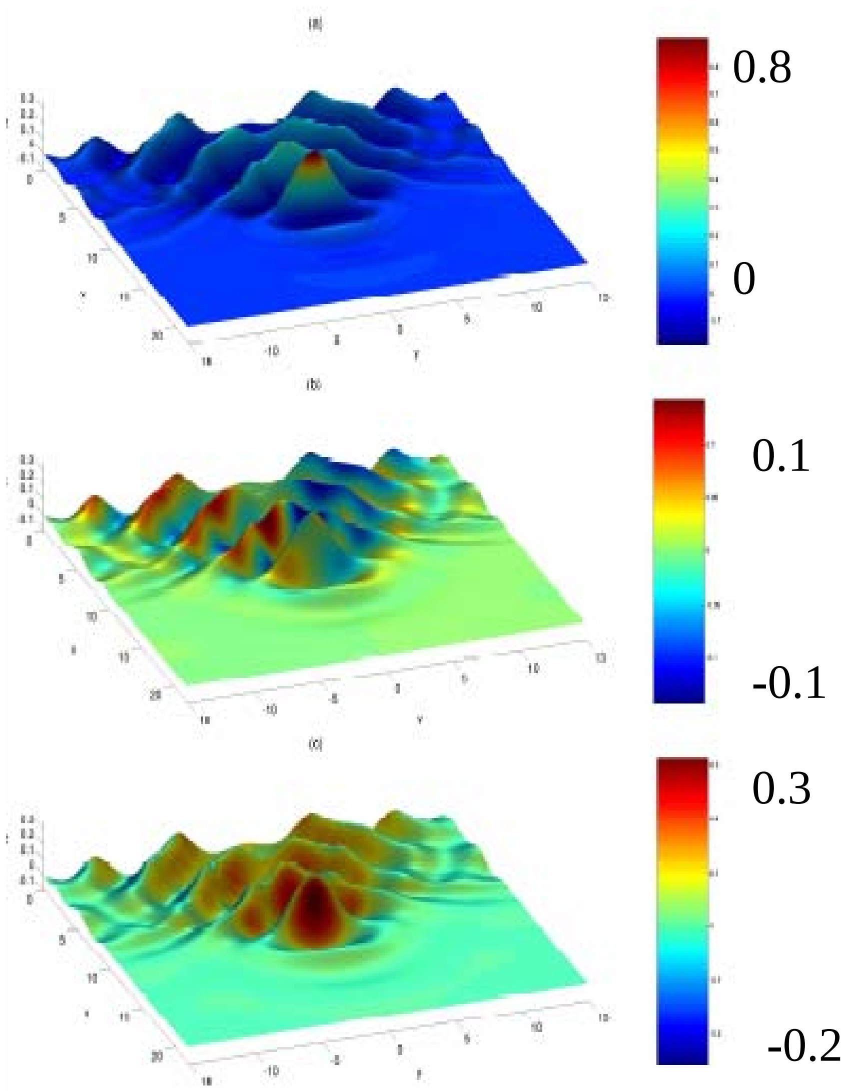
Internal velocities are shown



T = 33.76

Z = half way through the wave

Velocity fields on the free surface



Horizontal velocity

Transverse velocity

Vertical velocity

Time evolution of a 3D wave leading to breaking

Observations

* Circular trough located just in front of the wave (the so-called "hole in the sea" reported by freak wave eyewitnesses ?)

* Behind it, an even deeper trough has formed, separating the main wave from the curved crest line which follows it. This trough has a crescent shape.

* Strong asymmetry between the back and the front of the wave.

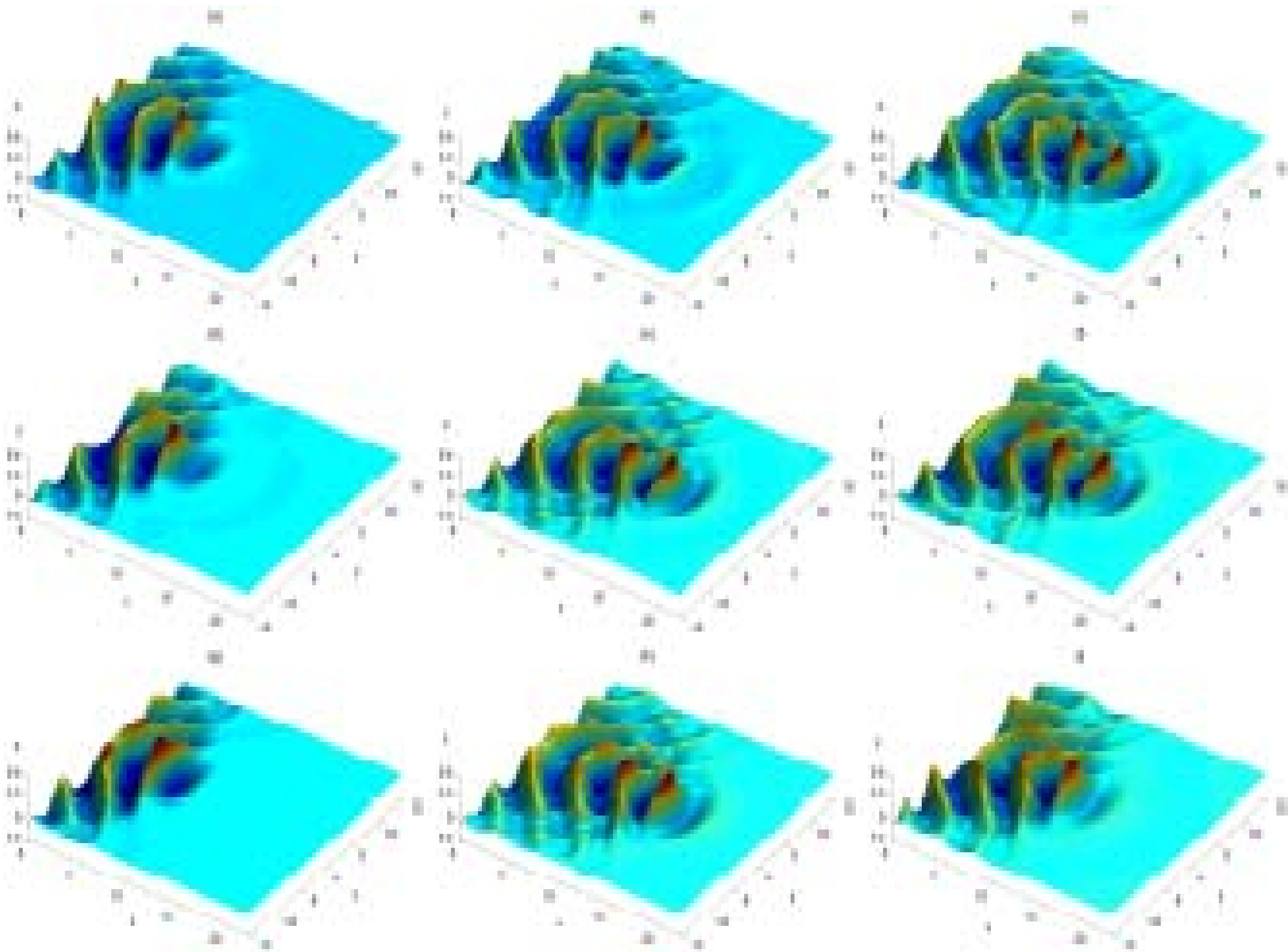
* The wave amplitude is significantly larger than that of the following waves which have not yet converged.

* This asymmetry increases with time and indicates that the wave is about to break. The wave itself appears like a curved front.

Animation #1

Animation #2

Various surface wave fields



d=1

d=2

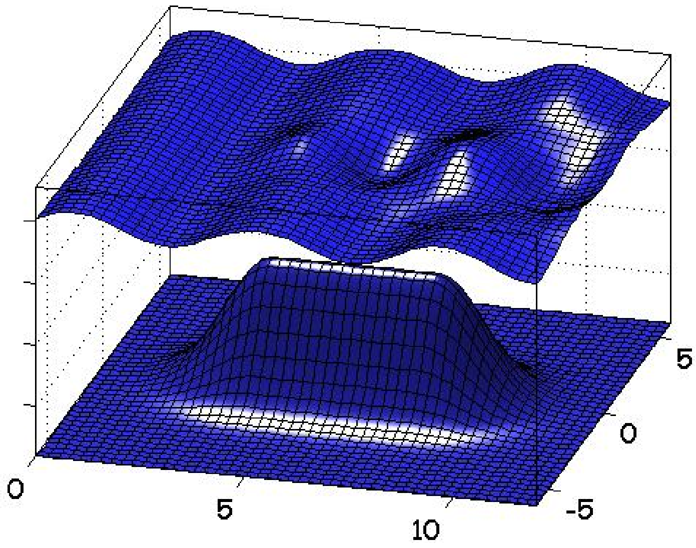
d=3

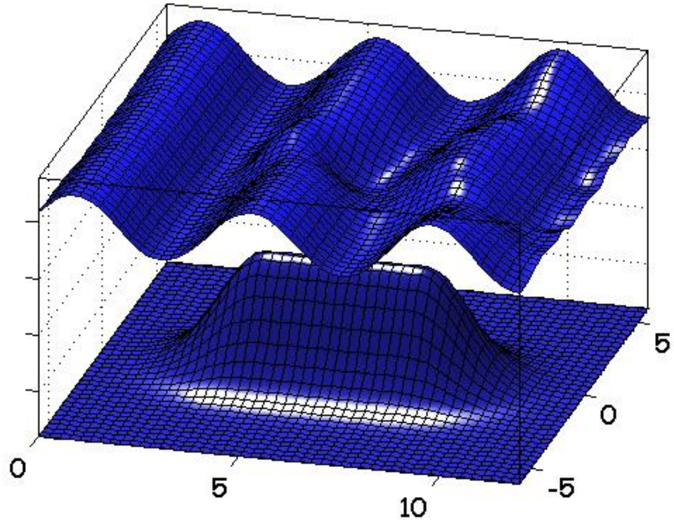
40 deg 45 deg 50 deg

Future work

Validation

- Experiments conducted by Kimmoun (First wave tank, Toulon)

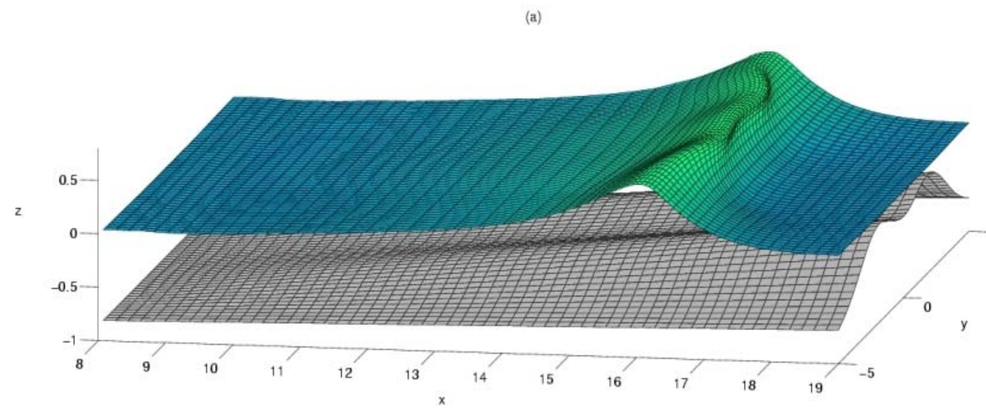




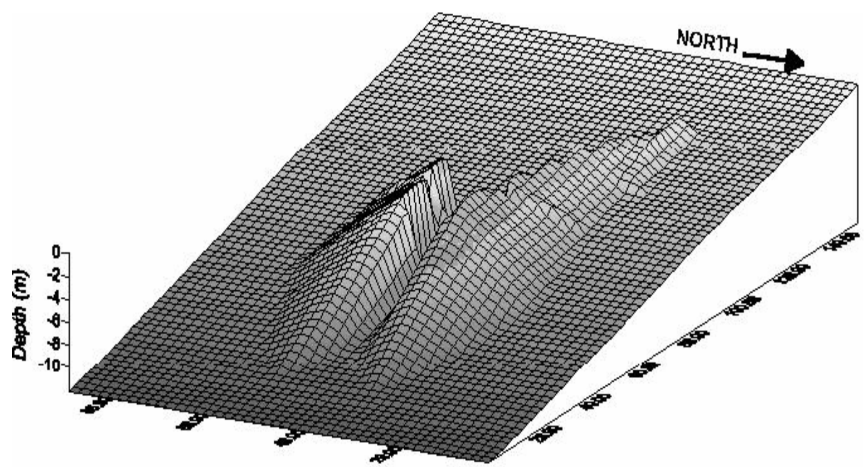
Applications and future work

Applications

- More and more complex bathymetries



Gold Coast Artificial Reef indicated by the two offshore breaking sections of wave in the foreground



Gold Coast Artificial Reef bathymetry