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Rogue waves as a result of modulational instability

If the spectral density of ocean wave is narrow in comparison with steepness, $\delta\omega / \omega < \mu \cong (ka)$, the modulation instability occurs. We have studied the nonlinear shape of this instability by performing massive numerical simulations of the Euler equations for ideal fluid with free surface. Our conclusion is the following - the modulational instability generates rogue waves.

We started with the exact Euler equation for potential flow of infinitely deep incompressible fluid in two dimensions and performed canonical map of the domain filled by fluid onto the lower half-plane. Under this map, the hydrodynamic equations are transformed to the elegant Dyachenko equations, which are suitable for numerical solution by implementation of the standard spectral code. We used the code with adjusting time step and adjusting number of spectral modes. This number varied within the limits $10^4/10^6$. All experiments were done in the standard wave tank of the length 2π.

In the first group of experiments we studied the development of modulational instability of slightly perturbed Stokes wave. Both steepness and wave numbers of the initial wave could be essentially varied. The most impressive results are obtained for the Stokes with wave number $n = 100$ and $\mu \cong 0.1$. In this case the modulational instability is a slow process. On the initial stage we observed exponential growth of perturbation followed by formation of a kind of one-dimensional turbulence. After more than ten inverse growth rates of the instability (two or more periods of initial wave) the process ends up by formation of the freak wave of an amplitude exceeding initial average level of surface elevation in four-five times. Onset of the freak wave is a catastrophic event taking only a few wave periods.

In the second group of experiments we studied the formation of freak waves from envelope quasisolitons. The quasisolitons of small steepness, $\mu < 0.1$, propagate peacefully and could be fairly approximated by the Nonlinear Schrodinger equation (NLSE). Quasisolitons of a moderate amplitude, $0.10 < \mu < 0.14$, still propagate without changing their form, however, they are essentially asymmetric. The quasisolitons of high steepness, $\mu > 0.14$, are unstable. Their evolution leads to a formation of freak wave. The freak waves appear also in collisions of relatively small amplitude solitons.

In the third group of experiments we generated stochastic quasi-monochromatic waves from the white noise by including into equations of the narrow-band weak instability (so far in the range of wave numbers 30/40). At a certain level of nonlinearity, the growth of instability is arrested by a sporadic wave breaking. On this background we saw rare events of rogue wave formation. We plan to accumulate enough amount of such events to determine the PDF of extreme wave appearance.

References:

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Abstract of talk given at ICMS Workshop on Rogue Waves December 2005.
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