



# Applying CVaR for Decentralized Risk Management of Financial Companies

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## **Abstract**

Over the past decade, financial companies have merged diverse areas including investment banking, insurance, retail banking, and trading operations. Despite this diversity, many global financial firms suffered severe losses during the recent recession. To reduce enterprise risks and increase profits, we apply a decentralized risk management strategy based on a stochastic optimization model. We extend the decentralized approach with the CVaR risk-metric, showing the advantages of CVaR over traditional risk measures such as Value at Risk. An example taken from the earthquake insurance area illustrates the concepts.

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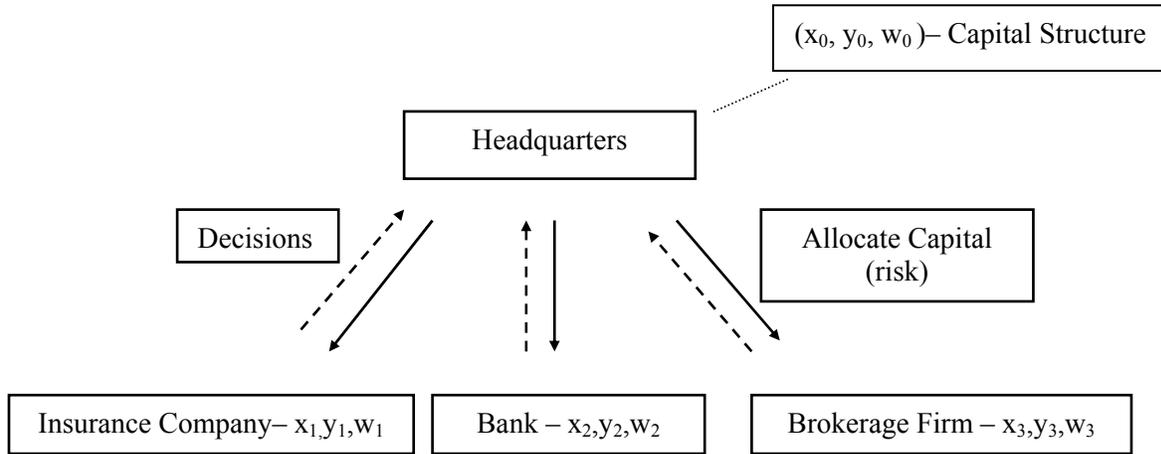
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## **1 Description of Centralized Optimization Model**

Large, global insurance companies can be managed via an integrated optimization framework to reduce enterprise risks and increase total profits. The topic of integrated risk management goes by several names, depending upon the application area. In banks and pension plans, it is referred as Asset and Liability Management, whereas it is called Enterprise Risk Management and Dynamic Financial Analysis (DFA) for non-financial and insurance companies, respectively. See Laster and Thorlacius (2000), Lowe and Stanard (1996), Mango and Mulvey (2000), Cariño et al. (1994), Mulvey et al. (2000) for applications of DFA. Also, see Boender (1997), Consigli and Dempster (1998) for applications in other financial domains.

In the insurance area, portfolio managers and underwriters require sophisticated analytical tools to achieve enterprise goals. For example, the insurance underwriter needs to understand the effects of adding an additional insurance account to the company's current portfolio (book) of activities. A developed decision support system, called SmartWriter, answers these questions for one application area, the property/catastrophe (P/C) business. SmartWriter employs data from earthquake and hurricane modeling systems to evaluate the effects of adding a new account(s) or subtracting an existing account(s) from the current portfolio. In addition, SmartWriter optimizes the portfolio composition to produce a portfolio meeting user-specified characteristics regarding risks and expected returns. See references Mulvey et al. (1998) for details, as well as Boender (1997), Kouwenberg (2001), Mulvey et al. (2000), and Cariño et al. (1998) for scenario-generation methods of economic factors/variables.

Managing a global insurance company via a centralized DFA system requires, however, the close coordination between the divisions (groups) and the company's headquarters. Mulvey et al. (2005) describes in detail a DFA system for the Towers Perrin Company and its Tillinghast business. This system has been implemented by global insurance companies, including AXA.



**Figure 1:** Coordination among headquarters and divisions - The headquarters decides on the capital allocation and the overall asset-liability management of the company.

To define the multi-divisional DFA model, assume  $D = \{1, \dots, d\}$  divisions, a single time period, and a set of scenarios ( $S = \{1, \dots, s\}$ ) depicting the uncertain quantities. To define the full set of decisions  $(x, y, w)$ , we introduce the variables below.

$x_d$  – vector of asset decisions related to division  $d \in D$ .

$y_d$  – vector of liability decisions related to division  $d \in D$ .

$w_d$  – vector of borrowing decisions related to division  $d \in D$ .

$(x_0, y_0, w_0)$  – enterprise-level decisions; the headquarters is defined as division  $d=0$ .

$c_d$  – capital allocated to division  $d \in D$  (total capital =  $C$ ).

$z_{(d)}$  – vector of statistics related to division  $d \in D$ . (e.g. profit, Value at Risk).

The property/casualty insurance area operates, in most cases, on an annual basis. Insurance contracts are issued for a single year at a time, with no guarantee of renewal. Hence, for simplicity, the DFA model consists of a single time-period in this paper. Mulvey et al. (2005) provides a comprehensive description of a full enterprise DFA for an insurance company.

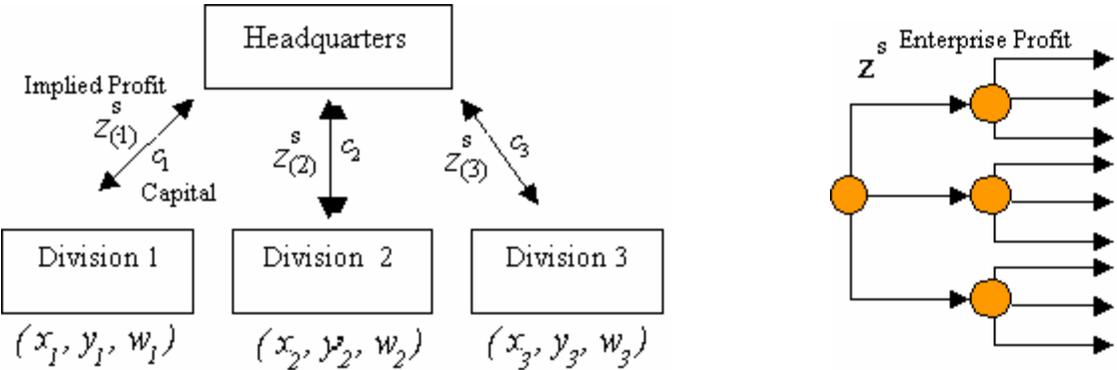
The overall objective is to maximize the company's shareholder value as defined by a multi-criteria utility function (with  $m$  objectives):

$$\text{Max } U [z_{1(0)}, \dots, z_{m(0)}] \quad (1)$$

where  $z_{n(0)} = g_{n(0)}(x_0, y_0, w_0)$  for  $n = 1, \dots, m$

and  $z_{1(0)} \dots z_{m(0)}$  describe the relevant enterprise-wide statistics for each objective, for example, the expected surplus at the end of planning period, the probability of credit downgrade over the next year, or the volatility of the profit/loss distribution. Other typical examples of objectives include the Value at Risk (VaR) at the end of the first year or the end of the planning period. The VaR objective is often employed by regulators and other stakeholders for financial applications. However, as we will see, there are problems with employing this function for optimizing the enterprise. A standard approach for DFA employs two objectives: the first  $z_{1(0)}$  is the expected enterprise profit and the second  $z_{2(0)}$  is an enterprise risk measure such as VaR.

The following scheme visualizes the centralized DFA model of a large-scale enterprise:



**Figure 2:** Centralized DFA Scheme

In centralized DFA, all major decisions are made within a unified planning environment. Thus, the company is able to deploy its resources in a company-optimal fashion. The impact of any new activity or of any change in existing activities can be immediately evaluated with respect to the enterprise. See Lowe and Stanard (1996) for a successful application of DFA for the Renaissance Reinsurance Company in Bermuda.

Unfortunately, the centralized approach is impractical for large, global organizations because of informational limits and complex local regulations within each country. Thus, the division’s optimal solution may be very different compared to the solution of the enterprise-wide optimization problem. The AXA insurance company depicts a global insurance company, with headquarters in Paris, France, for which a tightly managed, centralized DFA is difficult to implement. To come to a compromise among the divisions and the headquarters we introduce the decentralized approach to the DFA.

## 2 Formal Decentralized Optimization for DFA

The motivation behind introducing decentralized DFA is to design a practical system for analyzing and managing a large-scale global financial organization. Many, anticipatory scenarios are generated stochastically with the aim of providing information about the distribution of some important variables, like surplus or loss ratio at the division level.

The first step in planning for most global financial companies is to allocate capital to each division -- given  $c_d$  as the capital allocated to the division  $d$ , with constraint  $\sum c_d \leq C$ , where  $C$  is the enterprise capital. Typically, the headquarters decides on the capital allocated based on several factors. For instance, a division may be asked to compute its expected profit and volatility of profit based on historical values. Alternatively, the divisions may estimate their projected profit/loss in a deterministic fashion, based on a single scenario. To develop these values, the division must face its own asset and liability related decisions.

A simple approach would be to estimate the expected profit -  $E[z_{(d)}]$ ,  $\text{std}[z_{(d)}]$ , and correlation of  $(z_{(d)}, z_{(0)})$ . The typical structure today extends this approach where the divisions will be evaluated by means of a stochastic analysis. Namely once the capital is allocated, the divisions report to the headquarters their profit/loss estimates  $z_{(d)}^s$  under each scenario  $s \in S$ . Ideally, the decentralized DFA approach requires sending the profit/loss estimates between the divisions and the headquarters back and forth in an iterative scheme.

The critical issues in decentralized DFA arise in evaluating the performance of each division in a coordinated fashion. These issues can be adjusted with respect to the risk during the capital

allocation for example, by means of risk adjusted return on capital (RAROC). Following RAROC approach, the capital allocation decisions should be based on the perceived risk of each division as defined for instance, on the quantile estimation of the profit/loss distribution associated to each division (VaR). Taking cost of capital into account would maximize the shareholders' value and augment the riskiness of the division. A required return on the capital allocated above the riskless rate and risk adjusted profit calculations would also implement the RAROC approach to the optimization. Froot and Stein (1998) have developed a framework other than RAROC for analyzing the capital allocation and capital structure decisions facing financial institutions where they show how bank-level risk management considerations should factor into the pricing of the risks that cannot be easily hedged.

However, these approaches generally assume multinomial distributions for profits and other statistics. Unfortunately, this assumption is unrealistic in the P/C insurance domain since insurance losses possess extremely fat tails -- a very low probability of an enormous set of losses. This greatly underestimates the tail risk and hence the capital needs. And correlations are often unstable due to linkage with one or more underlying factors. Therefore, we should project future scenarios on an anticipatory, stochastic basis and calculate the implied profit  $z_{(d)}^s$  for division d under scenario s and  $z_{i(0)}^s$  enterprise profit under scenario s.

To develop a decentralized risk management system, we can turn to several alternative approaches. Perhaps, the most famous is the Dantzig and Wolfe (1961) method, which was originally intended to be a computational technique for solving large linear programs that have a special structure. The steps of the Dantzig-Wolfe algorithm can be interpreted as an economic

model for managing a distributed organization; see, for example, Baumol and Fabian, (1964). The master decision-maker (headquarters) must coordinate the solutions of the divisions to satisfy the corporate-wide constraint and maximize the enterprise objective function. The decomposition (Bradley et al., (1977)) is applied to problems with the structure where the constraints are divided into multiple groups. Usually the problem is much easier to solve if the complicating  $a_{ij}$  constraints are omitted, leaving only the easy subproblem constraints. Consider any subproblem solution – called a proposal. Given  $(x,y,w)$  (a feasible solution to the subproblem constraints), we may compute the amount of resource  $z_{n(0)}$  used in the  $n^{\text{th}}$  complicating constraint (objectives in our case), such as profit  $z_{1,(0)}$  associated with the proposal:

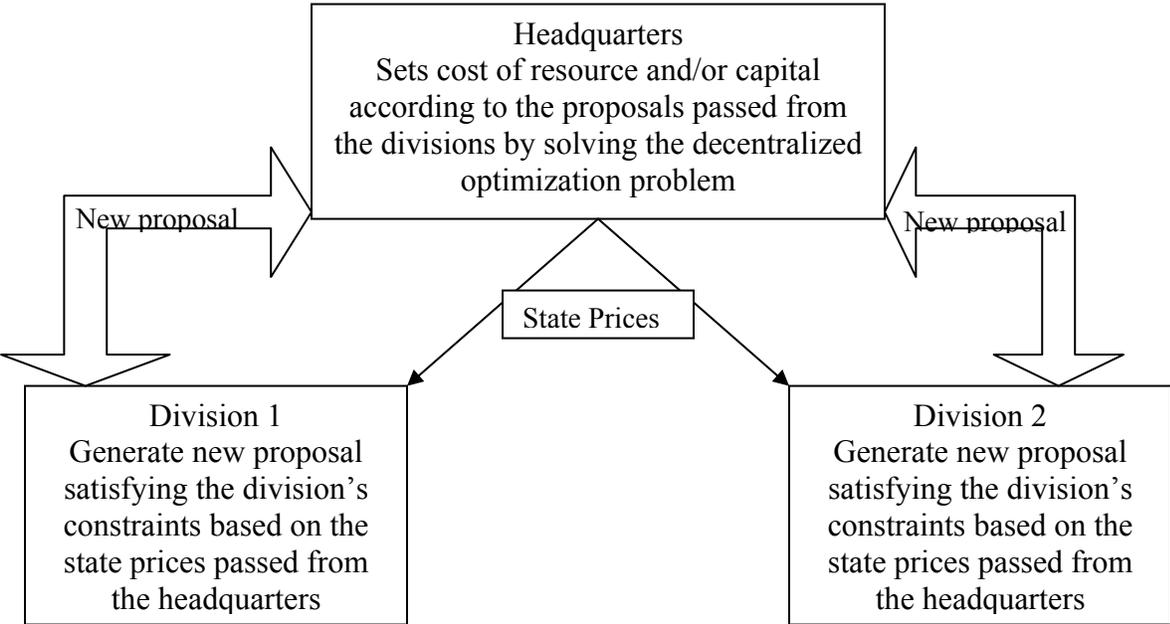
$$z_{n(0)} = g_{n(0)}(x_0, y_0, w_0) \quad \text{for } n = 1, \dots, m.$$

When  $k$  proposals to the subproblems are known, the procedure acts to weigh these proposals optimally. Decomposition in this context extends the interpretation to decentralized decision making. It provides a mechanism by which Lagrange multipliers (prices) can be used to coordinate the activities of multiple decision makers.

For our purposes to illustrate the concepts, we interpret the DFA problem as utility maximization for an insurance firm with two divisions. There are two levels of decision-making - headquarters and division. Subsystem constraints reflect the divisions' allocation of their own resources that are not shared. The complicating constraints limit corporate resources, which are shared and used in any proposal from either division. The main disadvantage of the centralized decision-making by optimizing the firm as a single entity arises because of the expense of gathering detailed information about the divisions in a form usable by either corporate headquarters or other

divisions. It is often best for each division and corporate headquarters to operate somewhat in isolation, having privacy and responsibility as much as possible.

In the decentralized approach where the decomposition algorithms can be applied, the information sent between the headquarters and the divisions are the *state-prices* (not just allocated capital), and *proposals*, from the divisions to the corporate coordinator. The state-price coordination is summarized in Figure 3. More will be said about this approach in the later sections.



**Figure 3:** State-Price Coordination among the headquarters and the divisions

### 3 Running the Two-Division Example

The purpose of this example is to illustrate the decentralized approach with a real-world application. As such, we have simplified the DFA model. Instead of employing multiple objectives as illustrated in (1), another approach would be to maximize the expected profit while incorporating a constraint on a risk measure such as VaR. However, it is difficult to optimize on

VaR since the problem becomes non-convex and this risk measure cannot be decomposed. Hence, as an approximation, we interpret the problem as a utility maximization of the certainty equivalent for an insurance firm with two divisions located in USA and Europe (US and EU). There are 216 accounts to invest and 50,000 scenarios for uncertain losses associated with each account. The loss data for each account is summarized by the SmartWriter system. There are two levels of decision-making – headquarters and division. This large-scale convex programming problem is solved and analyzed numerically by using the software LOQO. An extended analysis of the centralized and decentralized problem, and further decomposition results can be found in Mulvey and Erkan (2003) and Mulvey and Erkan (2004).

### 3.1 The Multi-Divisional Model

#### Indices:

$\text{acc}_{\text{US}}$ :  $\{1, 2, \dots, A_{\text{US}}\}$ , where  $A_{\text{US}}$  is the total number of accounts the US division can invest in  
(108 US accounts)

$\text{acc}_{\text{EU}}$ :  $\{1, 2, \dots, A_{\text{EU}}\}$ , where  $A_{\text{EU}}$  is the total number of accounts the EU division can invest in  
(108 EU accounts)

#### Parameters:

$l_j^s$ : loss matrix includes all the loss data regarding each account  $j \in (\text{acc}_{\text{US}} \cup \text{acc}_{\text{EU}})$  and  $s \in S$

$r_j$ : revenue generated from the investment in each account  $j \in (\text{acc}_{\text{US}} \cup \text{acc}_{\text{EU}})$

$C$ : initial starting capital (\$200,000)

$\rho_{\text{int}}$ : interest rate (for simplicity, all assets are invested in an interest bearing account.)

$\rho_{\text{bor}}$ : borrowing rate

$\theta_{bor}$ : the amount borrowed is bounded by  $\theta_{bor}^*$  maximum leverage (in the example we set maximum leverage equal to initial capital.)

$\theta_{US,acr}$ : asset to capital ratio of the US division

$\theta_{EU,acr}$ : asset to capital ratio of the EU division

$p_s$  : probability of scenario  $s$

$f_1$  and  $f_2$  : coefficients of the exponential part of the utility function

### **Variables:**

$x_{US}$  = initial assets of the US division

$x_{EU}$  = initial assets of the EU division

$c_{US}$  = amount of initial capital allocated to the US division

$c_{EU}$  = amount of initial capital allocated to in the EU division

$w_{US}$  = amount borrowed by the US division

$w_{EU}$  = amount borrowed by the EU division

$y_{US,j_{US}}$  = fraction invested in account  $j_{US} \in acc_{US}$  by the US division

$y_{EU,j_{EU}}$  = fraction invested in account  $j_{EU} \in acc_{EU}$  by the EU division

$z_{US}^s$  = US division's capital at the end of the investment horizon in each scenario

$z_{EU}^s$  = EU division's capital at the end of the investment horizon in each scenario

### **DFA Model:**

In (3), the objective function is the maximization of the expected utility of the firm's value (total capital) at the end of the horizon (see Bell (1995)). The utility function employed in this model is a combination of the linear and the negative exponential utility functions. Negative exponential

utility is sensible for an insurance enterprise since there is a probability of a negative total capital and this function has desirable theoretical properties, such as decreasing risk aversion. Different from the centralized model, we define separate variables for the initial asset, amount borrowed, ending capital with respect to the scenarios and fractions invested in each account regarding to both divisions. Especially in the reinsurance area, it is realistic to assume fractional investments since partial acceptances are possible. However contracts associated with other insurance businesses (such as homeowner insurance) would require binary variables ( $\{0 \text{ or } 1\}$ ) to identify the underwriting decisions.

The first group of constraints depicts the complicating constraints together with the associated state prices ( $\Pi$ 's). Note that the state prices are identified on a scenario by scenario basis. The second group of constraints defines the initial assets in terms of amount borrowed and revenues from accounts according to the fractions invested. The third group of constraints calculates the ending capital by taking the account losses with respect to the scenarios into account. The amount borrowed cannot exceed  $\theta_{\text{bor}}$  multiple of capital allocated to that division. The capital allocation should adapt to the initial capital of the firm. We indicate the upper/lower bounds for the amount borrowed in total and the fractions invested in each account. In the convex programming formulation of the decentralized model, each division decides on its own asset-liability management by monitoring the complicating resource constraints. Capital allocation is one of major outputs of the decentralized model.

$$\text{Maximize } \sum_{s \in S} p_s [z^s - f_1 \exp(-f_2 z^s)]$$

subject to:

$$z^s = z_{US}^s + z_{EU}^s \quad \forall s \in S \quad (\Pi_z^s)$$

$$c_{US} + c_{EU} = C \quad (\Pi_c)$$

$$0 \leq w_{US} + w_{EU} \leq \theta_{bor} C \quad (\Pi_w)$$

$$x_{US} = w_{US} + c_{US} + \sum_{j_{US} \in acc_{US}} r_{j_{US}} y_{US, j_{US}}$$

$$x_{EU} = w_{EU} + c_{EU} + \sum_{j_{EU} \in acc_{EU}} r_{j_{EU}} y_{EU, j_{EU}}$$

$$z_{US}^s = (1 + \rho_{int}) x_{US} - (1 + \rho_{bor}) w_{US} - \sum_{j_{US} \in acc_{US}} l_{j_{US}}^s y_{US, j_{US}} \quad \forall s \in S \quad (3)$$

$$z_{EU}^s = (1 + \rho_{int}) x_{EU} - (1 + \rho_{bor}) w_{EU} - \sum_{j_{EU} \in acc_{EU}} l_{j_{EU}}^s y_{EU, j_{EU}} \quad \forall s \in S$$

$$x_{US} \leq \theta_{US,acr} c_{US}$$

$$x_{EU} \leq \theta_{EU,acr} c_{EU}$$

$$0 \leq y_{US, j_{US}} \leq 1 \quad \forall a_{US} \in acc_{US}$$

$$0 \leq y_{EU, j_{EU}} \leq 1 \quad \forall a_{EU} \in acc_{EU}$$

$$0 \leq w_{US}$$

$$0 \leq w_{EU}$$

### 3.2 Convergence

As a major goal, the decentralized optimization model should give a similar optimal solution as the centralized optimization model. Moreover it has to answer all the questions such as the asset-liability decisions specific to both of the divisions and the optimal capital allocation to the divisions. Mulvey and Erkan (2003) showed that the decentralized model converges to the same optimal result as the centralized version.

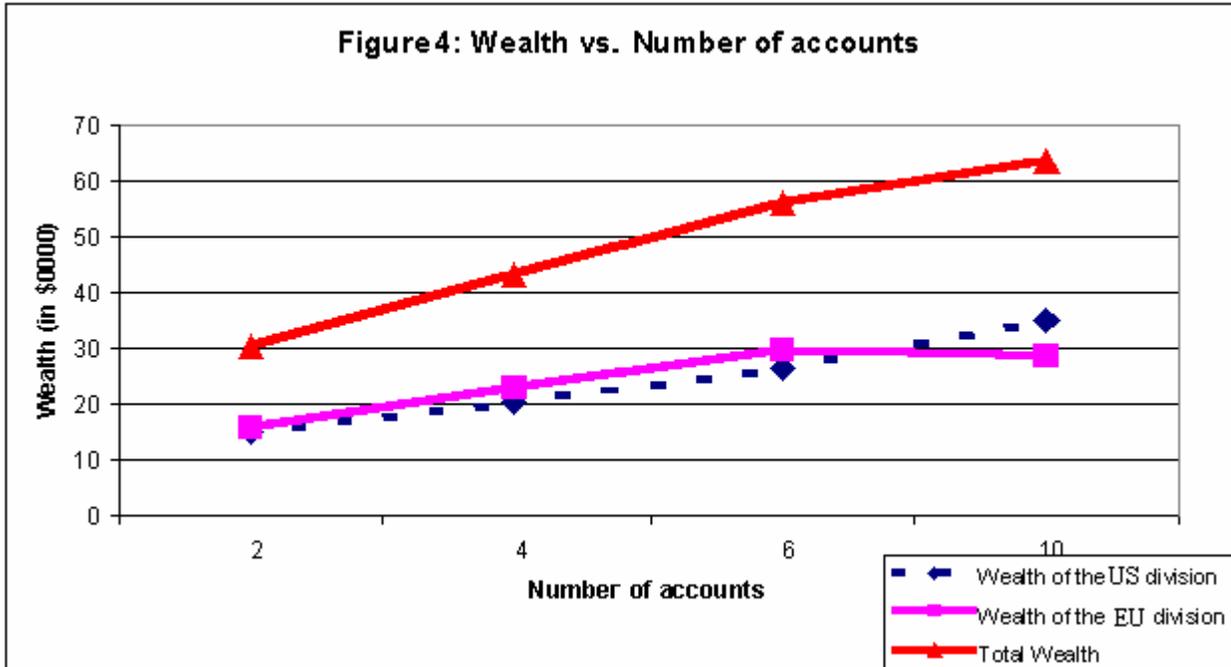
### **3.3 Numerical Experimentation**

In this section we illustrate the proposed approach via a numerical example. This application provides an ideal setting for decentralized risk management. First, the stochastic elements have been extensively analyzed by a number of large-scale efforts (e.g.: firms such as RMS, AIR, EQE International and others). Most insurance companies who sell catastrophe insurance run these scenario generators in order to compute the profit/loss distribution (Enz and Karl (2001)). This distribution is employed for capital allocation purposes and many other needs such as regulatory compliances by state insurance commissioners. Burket et al. (2001), Hoyland (1998), and Kaufman and Ryan (2000) give detailed discussions of the regulatory environment and other modeling complications for insurance companies.

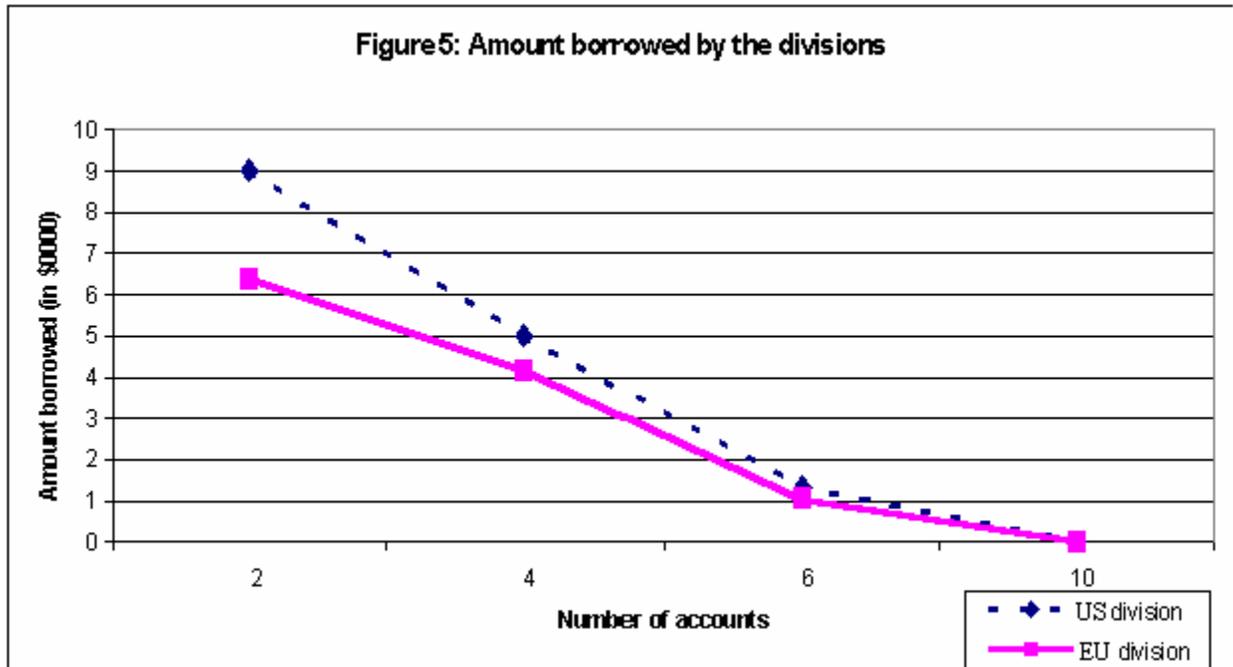
Second, clearly the tail insurance risks can be reduced by geographic diversification - earthquakes and hurricanes occur at specific physical locations. ). There is no contagion across CAT losses as occurs in financial markets. The tail characteristics are significant for a company since allocated capital, and a company's overall profitability is based on the anticipated tail risks (e.g. VaR). Thus, the advantages of decentralized risk management are clearly delineated.

#### **3.3.1 Addition of Accounts**

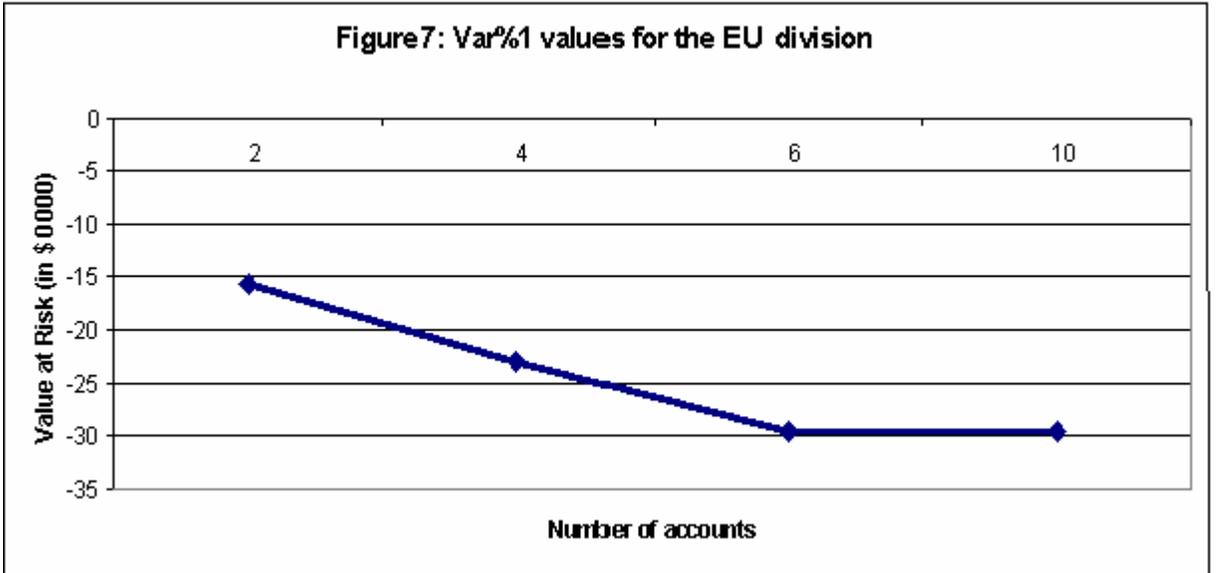
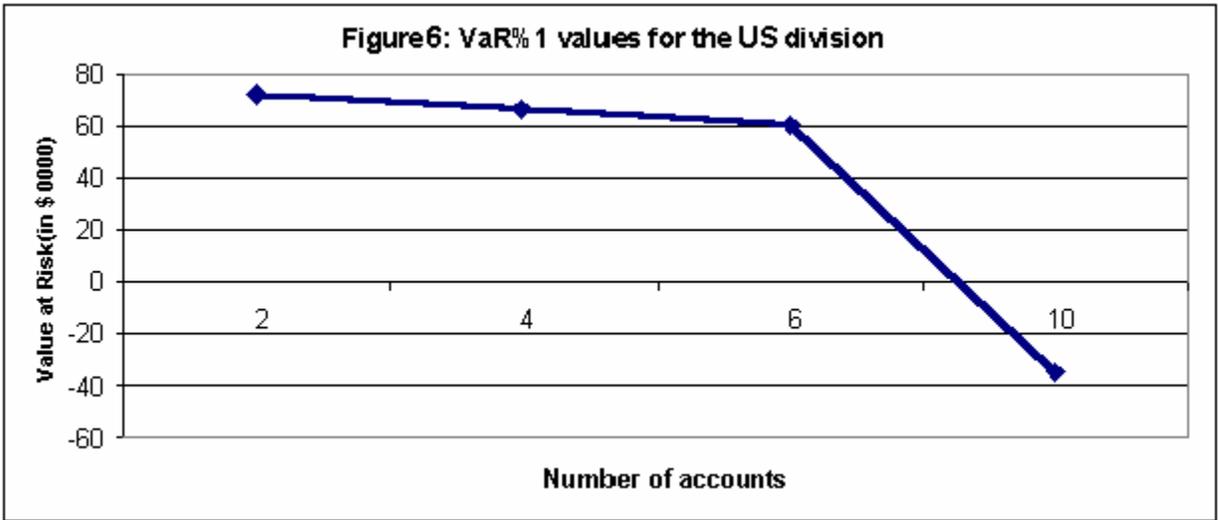
We focus on the decentralized model by analyzing the effects of adding new insurance accounts to the portfolio on the utility level, wealth/capital and risk statistics. We first evaluate the changes in the utility level and wealth with respect to account size.



The wealth structure of the US and the EU division is changing such that the total wealth of the company is increasing with a decreasing slope as more and more accounts are added to the portfolio. Meanwhile borrowing becomes less attractive for the divisions as the variety of the available accounts with different loss structures increases.



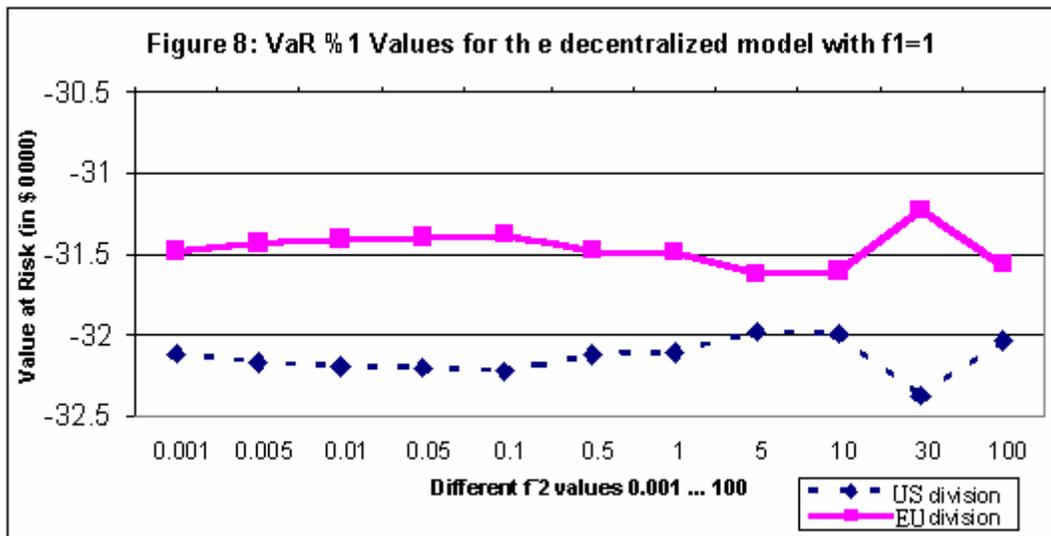
The decomposition of the aggregated initial assets for the decentralized model with 10 accounts looks exactly the same as in the centralized version (Mulvey and Erkan (2003)). This fact again assures that the decentralized optimization model will produce the same optimal results as the centralized model and moreover it will provide more information such as the optimum capital allocation, the mutually exclusive investment strategies of both divisions and the optimum amount the divisions should borrow.



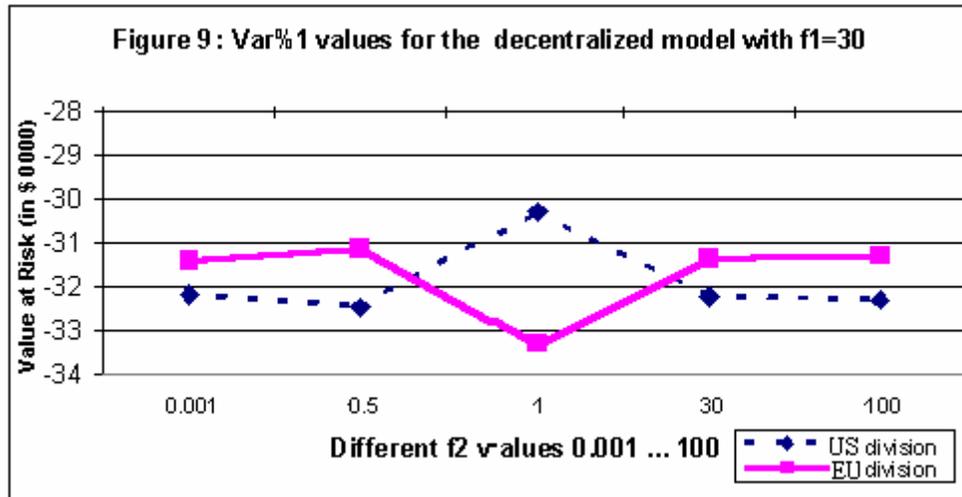
VaR is calculated based on the optimal output of the two-division example via the expected utility model. The VaR values are decreasing as the number of accounts increases. This pattern is similar to the VaR behavior in the centralized version.

### 3.3.2 Changing the Parameters $f_1$ and $f_2$

We have also analyzed the decentralized model with respect to different  $f_2$  values. We run the model first with  $f_1=1$  and  $f_2= 0.001, 0.005... 5, 30, 100$ . As  $f_2$  increases, we reduce the risk premium, thus becoming closer to risk neutrality. The pattern in Figures 8 and 9 is easy to interpret. Both divisions try to operate in a manner such that the aggregate company becomes more and more profitable. The increase in VaR of any division is compensated by the decrease in the VaR of the other division. In total, the VaR statistic for the whole company never increases, as  $f_2$  becomes larger.



When we run the decentralized model for different  $f_2$  values by fixing  $f_1=30$ , we have almost same type of trend in the VaR statistic. However one difference is that at  $f_1=30$  the pattern is just shifted leftwards since at a higher level of  $f_1$ , even small changes of  $f_2$  parameter have a much faster effect on the statistics of both divisions.



#### 4 Optimization Using Conditional Value-at-Risk (CVaR) as a Risk Measure

CVaR is a risk measure with significant advantages compared to VaR and is an excellent tool for risk management (see Rockafellar and Uryasev (2000) and also Rockafellar and Uryasev (2001)). It has a parallel in the insurance area as expected policyholder deficit (EPD) that uses expected loss as its base, expressing the target deficit as a percentage of expected loss. The paper by Mango and Mulvey (2000) discusses the merits and weaknesses of different risk measures for insurance companies. The reader interested in other applications of optimization techniques in finance area can find relevant papers in Ziemba and Mulvey (1998).

Artzner et al. (1999) presents and justifies a set of four desirable properties for measures of risk, and calls the measures satisfying these constraints “coherent”. Especially the sub-additivity property of CVaR makes this risk measure indispensable for decentralized risk management involving multiple divisions and headquarters.

CVaR is a more consistent risk measure than VaR. First, CVaR supplements the information provided by VaR and calculates the quantity of the excess loss. Since CVaR is greater than or equal to VaR, portfolios with a low CVaR also have a low VaR. Second, under quite general conditions, CVaR is a convex function with respect to positions (see Rockafellar and Uryasev (2000)), allowing the construction of efficient optimization algorithms. In particular, Rockafellar and Uryasev (2000) showed that CVaR can be efficiently minimized using linear programming techniques. A description of the approach for minimization of CVaR and the optimization problems with CVaR constraints can be found in Uryasev (2000).

The optimization problem with CVaR constraint for the insurance company is formulated below. In contrast to the VaR, CVaR is easy to apply in a decentralized setting. In addition, the optimization problem with CVaR is easy to solve; it also provides VaR as a byproduct, so that policy makers and other stakeholders can readily grasp the concepts. Again, we have separated the complicating (enterprise) constraints, with indications for the corresponding state prices ( $\Pi$ 's)

In the optimization model (4) below, the variable  $B$  designates the maximum tolerance set for the CVaR constraint. The optimization model is solved for different upper bounds on CVaR. By solving the model, we find the optimal investment strategy, the optimal capital allocation, corresponding VaR, which equals to the optimal  $\zeta^*$ , and the CVaR.

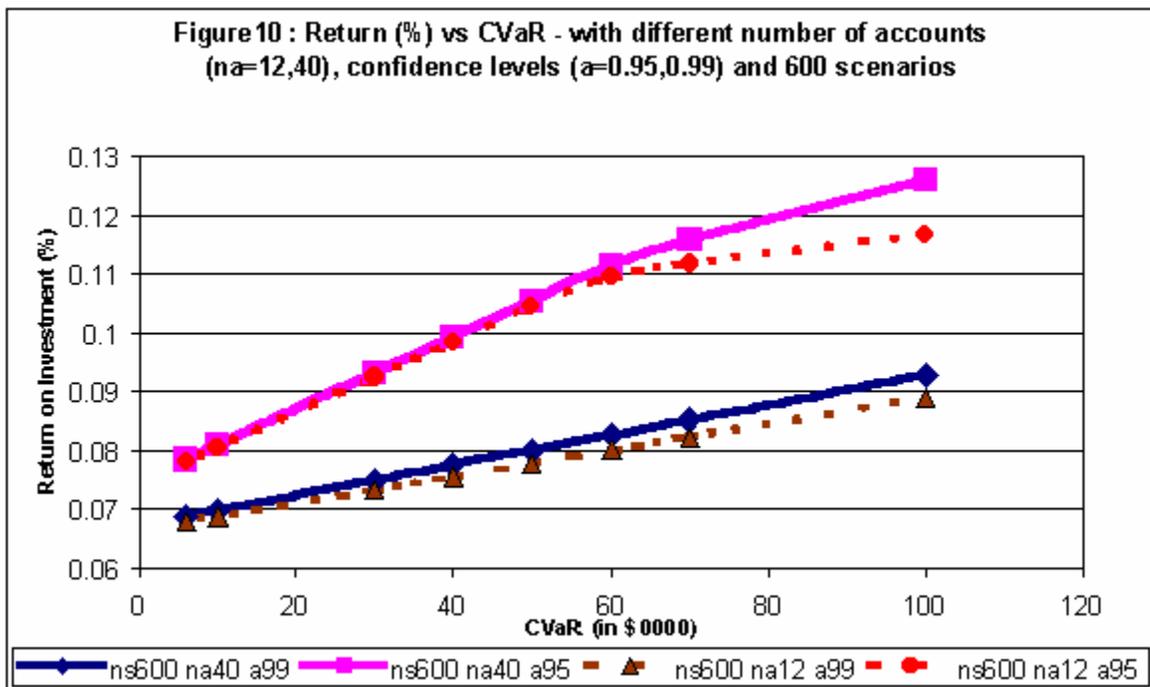
$$\begin{aligned}
& \text{Maximize} && \sum_s p_s z^s \\
& \text{subject to:} && \\
& \zeta + \nu \sum_{s \in \mathcal{S}} q_s \leq B && \\
& \nu = \frac{1}{((1-\alpha)ns)} \quad \text{where } ns := \text{total number of scenarios} && \\
& q_s \geq -z^s - \zeta \quad \forall s \in \mathcal{S} && \\
& q_s \geq 0 \quad \forall s \in \mathcal{S} && \\
& \dots\dots\dots && \\
& z^s = z_{US}^s + z_{EU}^s \quad \forall s \in \mathcal{S} \quad (\Pi_z^s) && \\
& c_{US} + c_{EU} = C \quad (\Pi_c) && \\
& 0 \leq w_{US} + w_{EU} \leq \theta_{bor} C \quad (\Pi_w) && \\
& \dots\dots\dots && \\
& x_{US} = w_{US} + c_{US} + \sum_{j_{US} \in acc_{US}} r_{j_{US}} y_{US,j_{US}} && \\
& x_{EU} = w_{EU} + c_{EU} + \sum_{j_{EU} \in acc_{EU}} r_{j_{EU}} y_{EU,j_{EU}} && \\
& \dots\dots\dots && \\
& z_{US}^s = (1 + \rho_{int})x_{US} - (1 + \rho_{bor})w_{US} - \sum_{j_{US} \in acc_{US}} l_{j_{US}}^s y_{US,j_{US}} \quad \forall s \in \mathcal{S} && (4) \\
& z_{EU}^s = (1 + \rho_{int})x_{EU} - (1 + \rho_{bor})w_{EU} - \sum_{j_{EU} \in acc_{EU}} l_{j_{EU}}^s y_{EU,j_{EU}} \quad \forall s \in \mathcal{S} && \\
& \dots\dots\dots && \\
& x_{US} \leq \theta_{US,acr} c_{US} && \\
& x_{EU} \leq \theta_{EU,acr} c_{EU} && \\
& 0 \leq y_{US,j_{US}} \leq 1 \quad \forall a_{US} \in acc_{US} && \\
& 0 \leq y_{EU,j_{EU}} \leq 1 \quad \forall a_{EU} \in acc_{EU} && \\
& 0 \leq w_{US} && \\
& 0 \leq w_{EU} &&
\end{aligned}$$

Similar to return-variance analysis, the efficient frontiers are constructed below for different numbers of available investment projects and for different  $\alpha$  (confidence level) values and the results are summarized in Table 1.

<b>Confidence Level = 0.95, Number of Accounts = 12</b>		
<b>B</b>	<b>Return (%)</b>	<b>VaR</b>
6	0.0783	-14.8419
10	0.0804	-13.9167
30	0.0925	-9.4781
40	0.0985	-6.8640
50	0.1045	-4.5364
60	0.1099	-0.4335
70	0.1118	9.3650
100	0.1167	27.2029
<b>Confidence Level = 0.99, Number of Accounts = 12</b>		
<b>B</b>	<b>Return (%)</b>	<b>VaR</b>
6	0.0679	-4.1903
10	0.0688	-1.6888
30	0.0733	10.8183
40	0.0755	17.0719
50	0.0778	23.3254
60	0.0800	29.579
70	0.0823	35.8325
100	0.0890	54.5933
<b>Confidence Level = 0.95, Number of Accounts = 40</b>		
<b>B</b>	<b>Return (%)</b>	<b>VaR</b>
6	0.0785	-14.2313
10	0.0809	-13.2065
30	0.0932	-8.0825
40	0.0994	-5.5205
50	0.1055	-2.9585
60	0.1116	-0.3750
70	0.1159	9.0059
100	0.1260	22.4572
<b>Confidence Level = 0.99, Number of Accounts = 40</b>		
<b>B</b>	<b>Return (%)</b>	<b>VaR</b>
6	0.0688	-4.5999
10	0.0698	-2.1586
30	0.0749	10.0471
40	0.0775	16.1501
50	0.0801	22.2531
60	0.0826	28.356
70	0.0852	34.459
100	0.0929	52.7679

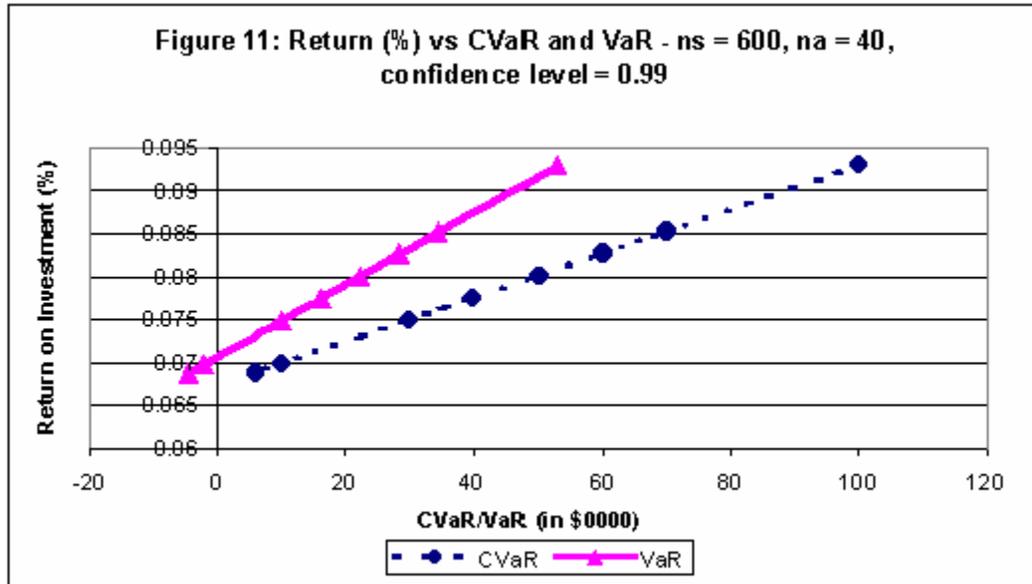
**Table 1:** Optimal Expected Return and VaR values for different upper bounds on the CVaR constraint in the optimization model (4) and at different confidence levels

Four different numerical experiments were undertaken. We solve the optimization model for two different confidence levels, namely 0.95 and 0.99. There are 600 scenarios for all of the experiments, and the procedure is repeated for 12 and 40 accounts. The upper bound for the constraint on conditional value-at-risk is taking the values 6, 10, 30, 40, 50, 60, 70 and 100 in terms of 10,000 US dollars consequently for each optimization problem. The return values are calculated for each upper bound, and the efficient frontiers are plotted in Figure 10.



In Figure 10, when we decrease the confidence level from 0.99 to 0.95, we are confronted with a higher return for the same risk measure, which is the upper bound on conditional value-at-risk in our case. On the other hand, increase in the account number has a positive effect for the returns with respect to the CVaR values. The effect of the increase in the number of available investment project on the returns at a given CVaR value is more dramatic for lower levels of confidence levels such as  $\alpha = 0.95$  than  $\alpha = 0.99$ . As the confidence level is decreased from 0.99 to 0.95, the

optimal investment strategy becomes less risk averse. Hence the advantage of having a variety of accounts can be exploited at a greater extent. The optimal VaR, the CVaR and the optimal return on investment are given in Figure 11 where we have a set of 600 scenarios and 40 accounts to invest and the alpha-level is set to 0.99.



## 5 Conclusions and Future Work

By coordinating its assets and liabilities at the enterprise level, a financial institution can increase overall profits *and* reduce risks. This coordination requires a large-scale, forward looking simulation. We showed that Dynamic Financial Analysis (DFA) within the property/casualty industry provides a systematic approach for diversifying the company's insurance activities. In fact, a global property/casualty insurance company has greater opportunities than a local company since the former can diversify across many regions and types of risks, thus reducing its capital requirements. However, a global insurance company is difficult to manage in a

centralized fashion due to complex local regulations/policies. The decentralized approach enables the divisions to increase flexibility and independence as compared with a centralized model.

The continuing convergence of the traditional capital and insurance markets should yield innovative approaches to managing emerging risks. Shareholders are increasingly holding boards of directors and senior executives to higher accountability standards. Aimed at giving shareholders more information and control and to increase the responsibility of directors, the Kon TraG bill was introduced into law in Germany in 1998. Especially the Kon TraG bill (Das Magazin fuer Risk Management, 07.2000) emphasizes the necessity of an early warning system in an enterprise. According to the law, German enterprises must possess viable risk management systems.

In the future, we plan to conduct research on the convergence properties of the decentralized algorithms under a variety of risk measures. As mentioned, the sub-additivity property of CVaR makes this risk measure indispensable for decentralized risk management involving multiple divisions and headquarters. The VaR measure fails in this regard since it is difficult to optimize an enterprise with VaR as an objective or constraint on enterprise risk.

Also, we intend to apply the concepts to firms outside the financial industry. The supply chain area, for example, could benefit by addressing uncertainties and disruptions via a stochastic optimization framework. Again, a decentralized approach has several advantages over a purely centralized model.

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